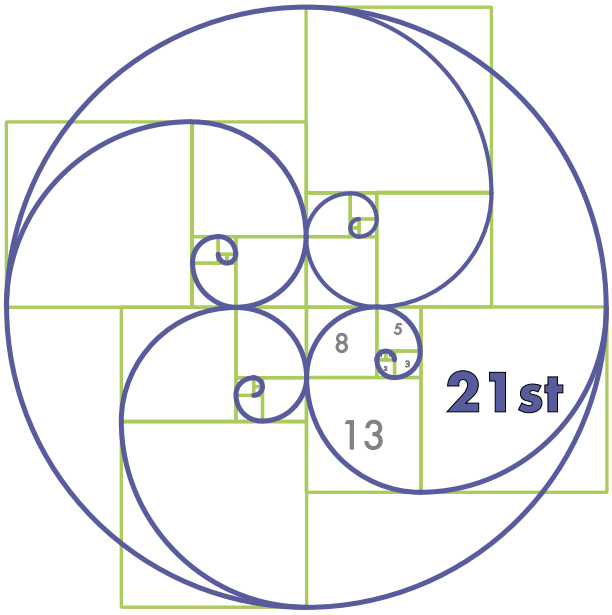

COMBINATORICS 2022

Mantua (IT)
May 30 – June 3, 2022

INdAM GNSAGA

UNIVERSITÀ di VERONA | Dipartimento di INFORMATICA

ICA The Institute of Combinatorics and its Applications

UNIMORE UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

UNIVERSITÀ DI PARMA

Fondazione UniverMantova

The printed version of this book of abstracts has been prepared on 5 May 2022. This online version has been last updated on 5th May 2022. We invite all the participants to check that their data are correct and up to date.

Overview

Presentation

The Combinatorics 2022 conference is organised by the research groups in *combinatorial geometry and design theory* of the University of Modena and Reggio Emilia, University of Parma and University of Verona. It takes place from May 30 to June 3 2022 in Mantua, in the rooms of the campus of Fondazione UniverMantova. This is the 21st edition of this conference held in Italy; originally it was scheduled for 2020, but it has been delayed due to the pandemic. The main themes of the conference are Finite and Incidence Geometries, Combinatorial Designs and Graph Theory.

Plenary speakers

- Herivelto Borges
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- Bence Csajbók
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- Nicola Durante
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- Gloria Rinaldi
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- *G.N.S.A.G.A. of INDAM* and the *Institute of Combinatorics and its Applications* for their constant sponsorship and support.
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  - the *University of Modena and Reggio Emilia*;
  - the *University of Parma*;
  - *Fondazione UniverMantova*;
- the *University of Brescia* which has provided the hosting resources for the site of the conference.

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Invited talks

The p-rank of curves of Fermat type

Herivelto Borges
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Joint work with: C. Gonçalves

Abstract

Let $\mathcal{X}$ be a nonsingular projective curve of genus $\mathfrak{g} > 0$, defined over an algebraically closed field $\mathbb{K}$ of characteristic $p > 0$. Arithmetic and geometric properties of $\mathcal{X}$ are often encoded in its birational invariants, some of which are the genus, the automorphism group, and the p-rank. The latter is the integer $\gamma(\mathcal{X})$, with $0 \leq \gamma(\mathcal{X}) \leq \mathfrak{g}$, such that $J[p](\mathbb{K}) \cong (\mathbb{Z}/p\mathbb{Z})^{\gamma(\mathcal{X})}$, where $J[p]$ is the kernel of the multiplication-by-p morphism on the jacobian J of $\mathcal{X}$. That is, $\gamma(\mathcal{X})$ is the number of copies of $\mathbb{Z}/p\mathbb{Z}$ in J, or equivalently, the number of independent unramified abelian p-extensions of the function field $\mathbb{K}(\mathcal{X})$.

The study of the p-rank is fundamental for a number of problems related to the classification of curves over finite fields. For instance, it is well-known that curves with somewhat large automorphism groups have zero p-rank. On the other hand, a conjecture by Guralnick and Zieve states that if $\mathcal{X}$ is ordinary, that is, $\gamma(\mathcal{X}) = \mathfrak{g}$, then $|\mathrm{Aut}(\mathcal{X})| \leq c\mathfrak{g}^{8/5}$. Other important topics, such as curves attaining the Hasse-Weil bound, can be naturally connected to the study of the p-rank.

In this talk, we discuss some problems and recent results regarding the characterization of the p-rank of certain families of curves. In particular, we will show how one can easily determine the p-rank of curves of type $y^m = x^n + 1$. In addition, we will present and discuss a few combinatorial challenges intrinsically related to our approach and results.

Functions over finite fields and their applications

Bence Csajbók

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MSC(2010): 11T06, 51E20, 51E21

Abstract

For a function $f : \mathbb{F}_q \to \mathbb{F}_q$, $q = p^n$, p prime, the graph of f is the affine point set $U_f = \{(x, f(x)) : x \in \mathbb{F}_q\}$. The set of directions determined by f is

$$\mathrm{dir}_f = \left\{ \frac{f(x) - f(y)}{x - y} : x, y \in \mathbb{F}_q,\ x \neq y \right\} \subseteq \mathbb{F}_q.$$

Extend $\mathrm{AG}(2, q)$ with ideal points and with the line at infinity ℓ_∞. For $d \in \mathbb{F}_q \cup \{\infty\}$ denote by (d) the ideal point associated to the parallel class of lines with slope d and define $D_f = \{(d) : d \in \mathrm{dir}_f\} \subseteq \ell_\infty$. The point sets U_f and D_f have been used to construct and characterise various combinatorially defined points sets of $\mathrm{PG}(2, q)$, such as Rédei type blocking sets, Korchmáros–Mazzocca arcs, ovals, semiovals, sets without tangents.

After a series of papers by Rédei, Megyesi, Ball, Blokhuise, Brouwer, Szőnyi, Storme, it is known that $|D_f| < \lfloor (q + 3)/2 \rfloor$ implies $f(x) = \alpha + g(x)$, where $\alpha \in \mathbb{F}_q$ and $g(x)$ is a linearised polynomial, i.e. $g(x) = \sum_{i=0}^{n-1} a_i x^{p^i}$. We will present a result which can provide useful information on U_f also when D_f is large, but some additional combinatorial properties are satisfied.

Let A and B denote an additive and a multiplicative subgroup of $\mathbb{F}_q$, respectively. Thanks to results by Carlitz, McConnel, Bruen, Levinger, the functions with the property $\mathrm{dir}_f \subseteq B$ are completely described. We will consider some related problems, such as $\mathrm{dir}_f \subseteq A$, $\mathrm{dir}_f \subseteq B \cup \{0\}$ and $\mathrm{dir}_f = \mathrm{dir}_h$, where f and h are linearised. Finally, we will turn our attention to linearised polynomials and present some matricial techniques with applications, e.g. in the proof of the completeness of certain small caps in $\mathrm{PG}(4n + 1, q)$, $q > 2$.

The talk is based on joint works with A. Aguglia, A. Cossidente, G. Marino, F. Pavese, O. Polverino, F. Zullo and Zs. Weiner.

Keywords: functions over finite fields, direction problem, linearised polynomial

σ-geometries of finite projective spaces

Nicola Durante

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Abstract

The sets of the absolute points of (possibly degenerate) polarities of a projective space are well known and deeply studied being (possibly degenerate) quadrics, symplectic spaces and Hermitian varieties.
We cannot say the same for the sets of the absolute points of (possibly degenerate) correlations, different from polarities. Apart from the planar non degenerate finite case that was studied and solved by B.C. Kestenband in 11 papers from 1990 to 2014, until 2019 nothing else was known.
In June 2019, the author gave a course [1] introducing the concept of a σ-quadric of a finite projective space, σ being an automorphism of the underlying field, starting the study of the sets of absolute points of a (possibly degenerate) correlation, different from a polarity, of a finite projective space. It turns out that the study of σ-quadrics is far more difficult than the study of quadrics, symplectic spaces and Hermitian varieties. Nevertheless, in some recent papers with co-authors J. D'haeseleer, G. Donati, G.G. Grimaldi σ-quadrics related to degenerate correlations, in spaces of low (at most 5) dimensions have been classified. They turn out to be related to: ovoids of quadrics, spreads of three dimensional projective space, flocks of cones, arcs, hyperovals of even order projective planes, translation planes and non linear MRD codes.
Moreover, σ-quadrics have also been the geometic object inspiring the idea of $(d, \boldsymbol{\sigma})$-Veronese varieties to the author, G. Longobardi and V. Pepe.

Keywords: correlations, sesquilinear forms, automorphism

MSC(2010): 05B25, 51E20

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Tensors in Finite Geometry

Michel Lavrauw

Sabanci University, Istanbul

Abstract

The concept of tensor products is ubiquitous in the scientific literature. The bulk of the research on such tensor products assumes the underlying field to be the real numbers $\mathbb{R}$ or the complex numbers $\mathbb{C}$. With the advancement of our knowledge and the development of new technology, the need for efficient algorithms to verify certain properties or compute numerical data from a given tensor has become a very popular research topic. In this talk we restrict our attention to the tensor product of a finite number of finite-dimensional vector spaces over a finite field. We will give a short introduction explaining the main concepts and research problems, and report on recent results in this area.

Generating Uniformly Distributed Random Steiner Triple Systems

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Abstract

A few methods for generating random Steiner triple systems (STSs) have been proposed in the literature, such as Cameron's algorithm (which is analogous to the Jacobson–Matthews algorithm for Latin squares) and Stinson's hill-climbing algorithm. To improve the understanding of these algorithms, an assessment is here carried out for STSs of both small and large orders. Some variants of Stinson's algorithm are further proposed. For large orders, the number of occurrences of certain configurations in the generated STSs are compared with the corresponding expected values of random hypergraphs. This is joint work with Daniel Heinlein.

On certain compositions and decompositions of configurations

Tomaž Pisanski

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Abstract

Compositions form a fundamental tool in the *synthetic approach* to configurations. They constitute a class of operations that turn input parameters into a new configuration. There are primarily two such classes of operations, depending on the nature of parameters. In case of integer parameters, the so-called *families of configurations* are constructed, eg. cyclic and polycyclic configurations, [2]. On the other hand, the operations of *Grünbaum calculus* are typical synthetic methods that compose large configurations from smaller ones [3, 5].

In the opposite direction, a *decomposition* takes a configuration and decomposes it into smaller incidence structures. Together with calculation of some *invariants*, decompositions constitute the *analytic approach* to configurations, [4, 7]. Both compositions and decompositions have been considered by various geometers already in the nineteenth century. For instance, configurations that may be viewed as *mutually inscribed multilaterals* were quite popular; for an early account on configurations, see [6].

In this talk we present the basic theory and give an overview of some compositions and decompositions that we used in our research, combined with some specific applications, *e.g. to splittability*, [1], that form our work in progress with several colleagues, mainly with Leah Berman and Gábor Gévay.

A partition of an (n_k) configuration into t subconfigurations is called a *t-decomposition of configuration of type* $(r, s), r \geq s$, if all parts have the same symbol (p_r, q_s) or complementary symbol (q_s, p_r). In this talk we restrict our attention to 2-decompositions of configurations. A particular case, when all parts are isomorphic or complementary is investigated in connection with the self-duality of the original configuration. 2-decompositions of 3-configurations and 4-configurations are presented in more detail. We present an algorithm for generating and analysing non-isomorphic configurations admitting certain 2-decompositions.

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Combinatorial Requirements for Large Scale Experimentation

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Joint work with: Charles J. Colbourn, John Stufken, and Yasmeen Akhtar

Abstract

Engineered networks play a critical role in our society. Some examples of such networks and their use include:

- the power grid for smart homes and cars,
- the Internet and wireless networks for education, business, and entertainment,
- transportation networks for the supply chain,
- green house gas monitoring networks using satellites,
- airborne drones and ground sensors across the world for supporting fighting forest fires,
- global networks instruments and data centers for scientific discovery,

and beyond.

The role of such networks is expected to continue to expand both in scale and scope, yet our understanding of them remains limited. These networks have evolved into complex systems with behaviours and characteristics that are beyond the characterizations and predictions possible using traditional modelling, analysis, and design approaches.

Experimentation is one method to understand engineered systems. The discipline of design and analysis of experiments provides a theoretical basis for experimentation with a long tradition of connections to algebra, geometry, and combinatorics.

While experimentation can be used for many purposes, our interest is in screening experiments, to identify the important factors that significantly impact the system responses. There are assumptions and limitations underlying many screening designs. Most designs only consider a set of factors that is "not too large," with two levels for each factor, and are not able to treat categorical factors. Further, the designs often miss unexpected behaviours resulting from cross-system interactions. In the analysis of the measurements gathered in the experiment, it is often assumed that the corresponding design is balanced, that the system reports in every run, the direction of a response is known for specific factors, and the data are normally distributed.

Overcoming such assumptions and limitations based on experimental needs of engineered networks lead to a number of challenging and novel problems in combinatorics, which we discuss.

Perfect 1-factorisations and hamiltonian Latin squares

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Abstract

A 1-*factorisation* of a graph is a decomposition of the edges of the graph into 1-factors (perfect matchings). The 1-factorisation is *perfect* if the union of any two of its 1-factors is a Hamilton cycle. Kotzig famously conjectured that the complete graph K_{2n} has a perfect 1-factorisation (P1F) for every positive integer n. Despite considerable attention, we are very far from proving this conjecture. Only 3 sparse infinite families have been constructed, together with some sporadic orders. Several exhaustive enumerations have recently revealed that there are 3155 P1Fs of K_{16}.

For P1Fs of the complete bipartite graph $K_{n,n}$ our state of knowledge is marginally better, with 7 sparse infinite families published. A P1F of $K_{n,n}$ is equivalent to a *row-Hamiltonian Latin square* of order n. These are Latin squares with no non-trivial Latin subrectangles; equivalently, the permutation which maps any row to any other row is an n-cycle. Each Latin square has six *conjugates* (also called parastrophes) obtained by uniformly permuting its (row, column, symbol) triples. Let $\nu(L)$ denote the number of conjugates of L that are row-Hamiltonian. It is easy to see that $\nu(L) \in \{0, 2, 4, 6\}$ and that $\nu = 0$ can be achieved for all $n > 3$. At the other extreme, $\nu = 6$ is achieved by the so-called *atomic* Latin squares, including the Cayley tables of cyclic groups of prime order. There is also a known infinite family with $\nu = 2$. We announce the first infinite family with $\nu = 4$. It allows us to build Latin squares in which *every* pair of rows form a Hamilton cycle and *no* pair of columns form a Hamilton cycle. As a corollary, we will answer a question on quasigroup varieties posed by Falconer in 1970.

Contributed talks

List of contributors

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Papillon graphs: perfect matchings, Hamiltonian cycles and edge-colourings in cubic graphs

Marién Abreu

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Joint work with: John Baptist Gauci; Domenico Labbate; Federico Romaniello; Jean Paul Zerafa

Abstract

A graph G has the Perfect-Matching-Hamiltonian property (is PMH) if for each one of its perfect matchings, there is another perfect matching of G such that the union of the two perfect matchings yields a Hamiltonian cycle of G. The study of graphs that are PMH, initiated in the $1970s$ by Las Vergnas and Haggkvist, combines three ¨ well-studied properties of graphs, namely matchings, Hamiltonicity and edge-colourings. In this talk, we present results for cubic graphs in an attempt to characterise those cubic graphs for which every perfect matching corresponds to one of the colours of a proper 3-edge-colouring of the graph. We discuss that this is equivalent to saying that such graphs are even-2-factorable (E2F), that is, all 2-factors of the graph contain only even cycles. The case for bipartite cubic graphs is trivial, since if G is bipartite then it is E2F. Thus, we restrict our attention to non-bipartite cubic graphs. A sufficient, but not necessary, condition for a cubic graph to be E2F is that it is PMH. We introduce an infinite family of non-bipartite cubic graphs, which we term papillon graphs, and determine the values of the parameters for which these graphs are PMH or are just E2F.

Keywords: Cubic graph, perfect matching, Hamiltonian cycle, 3-edge-colouring.

MSC(2010): 05C15, 05C45, 05C70

Stability of EKR Theorems in Circle Geometries

Sam Adriaensen

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MSC(2010): 05B25, 05C50, 05E30, 51E20

Abstract

Erdős-Ko-Rado (EKR) theorems deal with the classification of large intersecting families. Take an incidence structure with points and blocks, and call a subset $\mathcal{F}$ of the blocks an *intersecting family* if any two elements of $\mathcal{F}$ share a point. Then one wonders how large an intersecting family can be, and what structure intersecting families of large size must have. In most settings, the only way to construct intersecting families whose size is close to being maximal, is by fixing a point and taking all blocks through this point.

A fruitful approach to EKR problems is constructing a graph where the blocks are the vertices, and adjacency coincides with being disjoint. We want to prove that the only large cocliques in this graph are *canonical*, i.e. consist of blocks through a fixed point. Our strategy is using the expander mixing lemma to prove that an intersecting family $\mathcal{F}$ has either a small or a large intersection with a canonical coclique. Then we prove that the intersection of $\mathcal{F}$ with some canonical coclique is not small, hence large, which forces $\mathcal{F}$ to be a subset of this coclique.

This technique is applied in the context of circle geometries [1], which capture the behaviour of oval plane sections in $\mathrm{PG}(3,q)$ with quadrics containing at most one singular point. We prove that all blocks in an intersecting family of size at least $\frac{1}{\sqrt{2}}q^2+\mathcal{O}(q)$ contain a fixed point. We note that the number of blocks through a point is $q^2+\mathcal{O}(q)$.

Keywords: Erdős-Ko-Rado; Algebraic graph theory; Circle geometries.

References

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On certain quasi-Hermitian varieties and related questions

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Joint work with: Luca Giuzzi

Abstract

A two-character set in the projective space $PG(r,q)$ is a set S of n points with the property that the intersection number with any hyperplane only takes two values. The study of two-character sets is motivated by their connection to projective two-weight codes and (linearly represented) strongly regular graphs. If a two-character set V and a Hermitian variety in $PG(r,q^2)$ have the same intersection numbers with hyperplanes then V is a quasi-Hermitian variety. A Hermitian variety can be viewed as a classical quasi-Hermitian variety. In [1], non-classical quasi-Hermitian varieties $\mathcal{M}_{\alpha,\beta}$ of $\mathrm{PG}(r,q^2)$ depending on a pair of parameters α,β from the underlying field $\mathrm{GF}(q^2)$, were constructed. For $r=2$ these varieties are Buekenhout-Metz (BM) unitals, see [3]. We study the equivalence of the quasi-Hermitian varieties $\mathcal{M}_{\alpha,\beta}$ in $\mathrm{PG}(3,q^2)$ with q odd and show that they behave under this respect in a similar way as BM-unitals in $\mathrm{PG}(2,q^2)$; see [2]. Furthermore, we prove that the point-collinearity graph of $\mathcal{M}_{\alpha,\beta}$ is connected for $q\equiv 1 \pmod 4$ (which is the only interesting case) and, in this talk, we also point out the relationship between $\mathcal{M}_{\alpha,\beta}$ and certain secrete sharing schemes.

Keywords: Algebraic variety, Hermitian variety, unital

MSC(2010): 94B05, 51A05, 51E21

References

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q-Matroids and rank-metric codes

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Joint work with: Eimear Byrne

Abstract

In classical combinatorics, matroids generalize the notion of linear independence of vectors over a field. In this talk, we will introduce the concept of $\mathbb{F}_{q^m}$-independence of $\mathbb{F}_q$-spaces and we show that q-matroids generalize this notion. As a consequence, the independent spaces of a representable q-matroid will be defined as the $\mathbb{F}_{q^m}$-independent subspaces of the q-system associated to an $\mathbb{F}_{q^m}$-linear rank-metric code. Moreover, we will further investigate the link between codes and matroids.

Keywords: q-matroids; rank-metric codes; independence.

MSC(2010): 05B35, 11T71

Polynomials with maximal differential uniformity and the exceptional APN conjecture

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Joint work with: Fabien Herbaut and Ali Issa

Abstract

The differential uniformity of a polynomial $f \in \mathbb{F}_q[x]$ over a finite field $\mathbb{F}_q$ is defined by $\delta_{\mathbb{F}_q}(f) := \max_{(\alpha,\beta)\in\mathbb{F}_q^*\times\mathbb{F}_q} \sharp\{x \in \mathbb{F}_q \mid f(x+\alpha)-f(x) = \beta\}$.

Polynomials f such that $\delta_{\mathbb{F}_{2^n}}(f) = 2$ are highly relevant in cryptography and are called APN (for Almost Perfect Nonlinear). Polynomials which are APN over infinitely many extensions of $\mathbb{F}_2$ are called exceptional APN.

Extending results from [1] and [2], we prove that for infinitely many even degrees m, for n sufficiently large, all polynomials of degree m with a nonzero second leading coefficient have a maximal differential uniformity. This implies of course that such polynomials are not exceptional APN. For $m = 4e$ where e is a Gold number, this gives a contribution to the exceptional APN conjecture.

Theorem. Let $m = 2^r(2^\ell + 1)$ where $\gcd(r, \ell) \leq 2$ and $r \geq 2$ and $\ell \geq 1$. For n sufficiently large, for all polynomials $f = \sum_{k=0}^{m} a_{m-k}x^k \in \mathbb{F}_{2^n}[x]$ of degree m such that $a_1 \neq 0$ the differential uniformity $\delta(f)$ is maximal that is $\delta(f) = m - 2$. In particular, f is not exceptional APN.

Keywords: Differential uniformity; APN functions; Chebotarev theorem.

MSC(2010): 11T06, 11T71, 14G50

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On strongly regular graphs with 136 vertices

Robert F. Bailey

Memorial University (Canada) — Grenfell Campus

Joint work with: Alaina Pardy, Abigail Rowsell

Abstract

In this talk, we will consider strongly regular graphs with parameters $(136, 63, 30, 28)$. The best-known example of such a graph is $NO_8^-(2)$, a rank-3 graph arising from the group $PSO^-(8, 2)$. However, another graph with these parameters arises from a primitive action of the group $PSL(2, 17)$. We will see how this graph can be constructed, and how it differs from $NO_8^-(2)$ in many ways.

Keywords: Strongly regular graph; primitive group

MSC(2010): 05E30, 05C25, 20B25

The geometry of certain error-correcting codes

Simeon Ball

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Abstract

In this talk I will consider various error-correcting codes, including linear, additive and stabiliser codes. It is well known that if one considers the set of columns of a generator matrix of a linear code, then one can consider this set as a set of points in a finite projective space. The parameters of the code then translate over to properties of the point set. In this talk I will consider the geometry of various different types of codes and codes with certain nice properties, like Hermitian self-orthogonality. Most of the results I will mention are contained in the articles [1], [2], [3] and [1].

Keywords: Error-correcting codes, finite geometry

MSC(2010): 94B00, 11E39, 51E00

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On graphs whose spectral radius does not exceed $3/\sqrt{2}$

Francesco Belardo

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Abstract

The identification of (connected) graphs with "small" spectral radius of the adjacency matrix has been a quite investigated topic in Spectral Graph Theory. So far, the graphs whose spectral radius does not exceed $\sqrt{2+\sqrt{5}}$ are known. The next reasonable step is to look for those having the spectral radius between $\sqrt{2+\sqrt{5}}$ and $3/\sqrt{2}$. The authors of [2] proved that the structure is fairly simple: the vertices of maximum degree 3 appear exclusively either on a path or on a cycle, and such structure resembles the *quipus*. Here, we focus on the (so far) last characterization obtained for the analogous problem with respect to the signless Laplacian matrix and discuss its consequence on the adjacency case. The research has been conducted jointly with Maurizio Brunetti, Vilmar Trevisan and Jianfeng Wang.

Keywords: Graph Index, Graph Eigenvalues, Quipus

MSC(2010): 05C50

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On the number of rational points of curves over a surface in $\mathbb{P}^3$

Elena Berardini

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Joint work with: Jade Nardi

Abstract

In this talk, we will show that the number of rational points of an irreducible curve of degree δ defined over a finite field $\mathbb{F}_q$ lying on a surface S in $\mathbb{P}^3$ of degree d is, under certain conditions, bounded by $\delta(d+q-1)/2$. Within a certain range of δ and q, this result improves all other known bounds in the context of space curves. The method we used is inspired by techniques developed by Stöhr and Voloch [2]. In their seminal work of 1986, they introduced the Frobenius orders of a projective curve and used them to give an upper bound on the number of rational points of the curve. After recalling some general results on the theory of orders of a space curve, we will study the arithmetic properties of curves lying on a surface in $\mathbb{P}^3$, to finally prove the bound.

The talk is based on the preprint [1].

Keywords: algebraic curves, embedded surfaces, rational points, finite fields

MSC(2010): 11G20, 14G05, 14H50, 14J70

References

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The minimum degree of minimal Ramsey graphs for cliques

Anurag Bishnoi

TU Delft (Netherlands) — Department of Mathematics

Joint work with: John Bamberg and Thomas Lesgourgues

Abstract

We will present a new upper bound of $s_r(K_k) = O(k^5 r^{5/2})$ on the Ramsey parameter $s_r(K_k)$ introduced by Burr, Erdős and Lovász in 1976, which is defined as the smallest minimum degree of a graph G such that any r-colouring of the edges of G contains a monochromatic K_k, whereas no proper subgraph of G has this property. This improves the previous upper bound of $s_r(K_k) = O(k^6 r^3)$ proved by Fox et al. The construction used in our proof relies on a group theoretic model of generalised quadrangles introduced by Kantor in 1980.

Keywords: Ramsey graphs; Generalized quadrangles; Heisenberg groups; Packings

MSC(2010): 05D10, 05C55, 51E12

References

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Edge balanced star-hypergraph designs and vertex colorings of path designs

Paola Bonacini

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Joint work with: Lucia Marino

Abstract

Let $K_v^{(3)} = (X, \mathcal{E})$ be the complete hypergraph, uniform of rank 3, defined on a vertex set $X = \{x_1, \dots, x_v\}$, so that $\mathcal{E}$ is the set of all triples of X. Let $H^{(3)} = (V, \mathcal{D})$ be a subhypergraph of $K_v^{(3)}$, which means that $V \subseteq X$ and $\mathcal{D} \subseteq \mathcal{E}$. We call 3-*edges* the triples of V contained in the family $\mathcal{D}$ and *edges* the pairs of V contained in the 3-edges of $\mathcal{D}$, that we denote by $[x, y]$. A $H^{(3)}$-design Σ is called *edge balanced* if for any $x, y \in X$, $x \neq y$, the number of blocks of Σ containing the edge $[x, y]$ is constant. We consider the star hypergraph $S^{(3)}(2, m+2)$, which is a hypergraph with m 3-edges such that all of them have an edge in common. We completely determined the spectrum of edge balanced $S^{(3)}(2, m+2)$-designs for any $m \geq 2$, that is the set of orders v for which such a design exists. Then we consider the case $m = 2$ and we denote the hypergraph $S^{(3)}(2, 4)$ by $P^{(3)}(2, 4)$. Starting from any edge-balanced $S^{(3)}(2, \frac{v+4}{3})$, with $v \equiv 2 \mod 3$ sufficiently big, for any $p \in \mathbb{N}$, $\lceil \frac{v}{2} \rceil \leq p \leq v$, we construct a $P^{(3)}(2, 4)$-design of order $2v$ with feasible set $\{2, 3\} \cup [p, v]$, in the context of proper vertex colorings such that no block is either monochromatic or polychromatic.

Keywords: design, edge balanced, hypergraph, vertex coloring

MSC(2010): 05C15, 05B05

References

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Saturating systems in the rank metric

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Joint work with: Martino Borello, Eimear Byrne

Abstract

A set $\mathcal{S} \subseteq \mathrm{PG}(k-1,q)$ is called *ρ-saturating* if for any point $Q \in \mathrm{PG}(k-1,q)$ there exist $\rho+1$ points $P_1,\dots,P_{\rho+1} \in \mathcal{S}$ such that $Q \in \langle P_1,\dots,P_{\rho+1}\rangle$ and ρ is the smallest value with this property.

Recall that the covering radius of an $[n,n-k]_q$ code is the least integer ρ such that the space $\mathbb{F}_q^n$ is covered by spheres of radius ρ centered on codewords. It is well-known that a linear $[n,n-k]_q$ code endowed with the Hamming metric has covering radius ρ if every element of $\mathbb{F}_q^k$ is a linear combination of ρ columns of a generator matrix of the dual code (that is the orthogonal space with respect to the standard inner product), and ρ is the smallest value with such a property. The correspondence between projective systems and linear codes w.rt. the Hamming metric specializes to a correspondence between $(\rho-1)$-saturating sets of size n in $\mathrm{PG}(k-1,q)$ and the dual of $[n,n-k]_q$ codes of covering radius ρ.

In this talk, we will discuss a generalization of saturating sets, called *saturating systems*, that extend this correspondence to rank-metric codes. We will analyse the properties of such geometrical objects and give some explicit constructions.

Keywords: Rank-metric codes, covering radius, linear systems

MSC(2010): 94B27, 11T71, 14G50

Small strong blocking sets by concatenation

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Joint work with: Daniele Bartoli

Abstract

Strong blocking sets and their counterparts, minimal codes, attracted lots of attention in the last years. Combining the concatenating construction of codes with a geometric insight into the minimality condition, we explicitly provide infinite families of small strong blocking sets, whose size is linear in the dimension of the ambient projective spaces.

In this talk, we will first introduce the objects and their connections and secondly we will present the constructions.

Keywords: Strong blocking sets; minimal codes; concatenation.

MSC(2010): 94B05, 51E21, 51E20

References

[1] D. Bartoli, M. Borello. "*Small strong blocking sets by concatenation*", arXiv:2109.00584, 2021.

Minimal blocking sets in small Desarguesian projective planes

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Joint work with: prof. Kris Coolsaet and prof. Veerle Fack

Abstract

All minimal blocking sets (up to equivalence) in Desarguesian projective planes of order ≤ 9 were generated by computer. These blocking sets were then classified according to size of the set, and order of the projective automorphism group and collineation group. Explicit descriptions or constructions are given for some sets, in particular (but not exclusively) for those blocking sets with a fairly large automorphism group. Some of these constructions can be generalised to Desarguesian projective planes of higher order.
We have found $\mathrm{PG}(2,7)$ to have 1433 inequivalent minimal blocking sets; $\mathrm{PG}(2,8)$ has over 45 thousand and $\mathrm{PG}(2,9)$ has over 15 million minimal blocking sets (inequivalent under $\mathrm{PL}(3,8)$ and $\mathrm{PL}(3,9)$, respectively). We have been able to describe several of these sets as unions of orbits of powers of Singer cycles. Other minimal blocking sets are related to orbits of $\mathrm{Sym}(4)$ and $\mathrm{Sym}(5)$, sum-free sets, the Hessian configuration, algebraic curves and unions of Fano subplanes.

MSC(2010): 51E21

Keywords: Blocking Sets; Desarguesian Projective Planes

Toward a solution of the Hoffmann Program for signed graphs

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Joint work with: Francesco Belardo

Abstract

Let G be a simple graph, and let $M(G)$ be any complex-valued matrix associated to G in a prescribed way. The M-spectral radius $\rho_M(G)$ of G is the largest norm of its M-eigenvalues. A real number $\gamma(M)$ is said to be an *M-limit point* if there exists a sequence of graphs $\{G_k \mid k \in \mathbb{N}\}$ such that $\rho_M(G_i) \neq \rho_M(G_j)$ whenever $i \neq j$ and $\lim_{k\to\infty} \rho_M(G_k) = \gamma(M)$. One of the two sides of the *Hoffman program with respect to* M consists in determining all the possible values for the M-limit points. We investigate and solve this part of the Hoffman program for M being the adjacency matrix of signed graphs, i.e. graphs whose edges have a positive or a negative sign.

Keywords: Signed graphs, Hoffman limit points, spectral radius.

MSC(2010): 05C22, 05C50

References

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A Spot for Strong Difference Families

Marco Buratti

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Abstract

A design with an automorphism group $(G, +)$ acting sharply transitively on the points is said to be G-regular. So, up to isomorphism, its point set is G and any translate $B + g$ of any block B is a block as well. Sometimes the construction of a $(G \times H)$-regular 2-design with H the additive group of a suitable ring can be greatly facilitated by cleverly choosing the projections of its blocks on G in advance. This is the idea hidden in the notion of a *strong difference family* (SDF) implicitly used by many authors for more than one century (see, e.g., [2]) but formally introduced for the first time in [1]. Since then, SDFs have been successful in the construction of many new infinite series of designs (see, e.g., [1, 3]) which cannot be surveyed in a short talk. I will settle for advertising SDFs by selecting one of my most recent results achieved with their use.

Keywords: 2-design; automorphism group; difference family.

MSC(2010): 05B05, 05B10

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Maximal Unrefinable Partitions into Distinct Parts

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Joint work with: Riccardo Aragona, Roberto Civino, Massimo Lauria

Abstract

A partition into distinct parts is refinable if one of its parts a can be replaced by two different integers which do not belong to the partition and whose sum is a, otherwise is unrefinable. For example, the partition $(1, 2, 5, 7)$ is refinable because we can write 7 as $3 + 4$, while the partition $(1, 2, 3, 6, 7, 11)$ is unrefinable.

Clearly, unrefinability is a non-trivial limitation on the distribution of the parts. For this reason, we decided to focus on the size of the largest element, for which we found an upper bound, and on the number of partitions which reach the bound, that we call maximal.

We start our study with the easier problem of counting maximal unrefinable partitions of the triangular number $T_n = \frac{n(n+1)}{2}$. We prove that, if n is even, then there exists only one maximal unrefinable partition of T_n, while if n is odd we show a one-to-one correspondence between the maximal unrefinable partitions of T_n and the partitions in distinct parts of k, where $n = 2k - 1$.

If time permits, I will present a generalization of the framework introduced to study the case of T_n to the case of any integer. Also in this case we obtain a complete classification of maximal unrefinable partitions and we show, again, that this is related to suitable partitions into distinct parts.

Keywords: Unrefinable Partition, Partition into Distinct Parts, Triangular numbers, bijective proof

MSC(2010): 11P81, 05A17, 05A19

References

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Combinatorial identities of the Rogers-Ramanujan type

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Joint work with: A. Meurman, M. Primc

Abstract

We conjecture combinatorial Rogers-Ramanujan type colored partition identities related to standard representations of the affine Lie algebra of type $C_\ell^{(1)}$, $\ell \geq 2$, and we conjecture similar colored partition identities with no obvious connection to representation theory of affine Lie algebras.

Keywords: Integer partitions; Affine Lie algebras;

MSC(2010): 05A17, 05A19

References

[1] S. Capparelli, A. Meurman, M. Primc. "*New Combinatorial identities of the Rogers-Ramanujn type*", Preprint, https://arxiv.org/abs/2106.06262

Frustration-critical signed graphs

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Joint work with: Eckhard Steffen

Abstract

A signed graph (G, Σ) is a graph G together with a set $\Sigma \subseteq E(G)$ of negative edges. A circuit is positive if the product of the signs of its edges is positive. A signed graph (G, Σ) is balanced if all its circuits are positive. The frustration index $l(G, \Sigma)$ is the minimum cardinality of a set $E \subseteq E(G)$ such that $(G - E, \Sigma - E)$ is balanced [2, 3]. A signed graph (G, Σ) is k-critical if $l(G, \Sigma) = k$ and $l(G - e, \Sigma - e) < k$, for every $e \in E(G)$.

We present decomposition and subdivision of critical signed graphs and completely determine the set of t-critical signed graphs, for $t \leq 2$. Critical signed graphs are characterized. We then focus on non-decomposable critical signed graphs. In particular, we characterize the set S^* of non-decomposable k-critical signed graphs not containing a decomposable t-critical signed subgraph for every $t \leq k$. We show that S^* consists of cyclically 4-edge-connected, projective-planar cubic graphs. Furthermore, we construct k-critical signed graphs of S^* for every $k \geq 1$ [1].

Keywords: Signed graphs; frustration index; criticality.

MSC(2010): 05C22, 05C75

References

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[2] Dorwin Cartwright and Frank Harary. "*Structural balance: a generalization of Heider's theory*", Psychological Review, **63**:277–293, 1956.

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A spectral theory for gain graphs

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Joint work with: D. D'Angeli, A. Donno

Abstract

Gain graphs are graphs whose oriented edges are labeled with elements of a group, in such a way that to opposite orientations correspond inverse elements. They are a generalization of *signed graphs*, which are graphs whose edges can be positive or negative. The *spectral theory of graphs* has a very natural generalization to signed graphs since Hermitian matrices are naturally associated with a signed graph. A seminal result is a characterization of *balanced signed graphs* (graphs where the product of the signs of the edges along any cycle is positive) in terms of their spectra.

The balance has an natural generalization to general gain graphs, while the *spectral theory of gain graphs* has been well studied only for special groups, since matrices that come out with a gain graph on an abstract group G are not complex, but group algebra valued matrices. Our approach is to use a unitary representation π of the group of gains in order to obtain a Hermitian matrix and therefore a real spectrum of the gain graph.

In this talk I will show how this method actually recover many special cases and how it can be used to generalize results from the classic case (e.g., the aforementioned balance result). Moreover, I will discuss how the choice of representation can affect the spectrum and the notion of *cospectrality*.

MSC(2010): 05C22, 05C25

Keywords: Spectral graph theory; gain graphs; signed graphs; adjacency matrix; balance

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An algorithmic method to compute plat-like Markov moves for genus two 3-manifolds

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Abstract

The talk deals with equivalence of links in 3-manifolds of Heegaard genus 2. Starting from a description of such a manifold introduced in [1], that uses 6-tuples of integers and determines a Heegaard decomposition of the manifold, we construct an algorithm (implemented in c++) which allows to find the words in $B_{2,2n}$, the braid group on 2n strands of a surface of genus 2, that realizes the plat-equivalence for links in that manifold. In this way we extend to the case of genus 2 the result obtained in [2] for genus 1 manifolds. We describe in particular a case of this construction.

Keywords: Knots; 3-manifolds; equivalence; Heegaard diagrams; 6-tuples; algorithms

MSC(2010): 57K10, 57M15, 57-04, 05C99

References

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Linear equations for the weight distribution of codes over finite chain rings

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Joint work with: Alessio Meneghetti

Abstract

The determination of the weight distribution of a code is of both theoretical and practical interest. A question naturally arising in the context of algebraic codes is whether we can take advantage of the underlying algebraic structure to obtain some information useful to the computation of the weight distribution.

In our work we focus on linear codes over finite chain rings. For such codes we determine new linear equations for their weight distribution by counting the number of some special submatrices of the parity-check matrix.
Let C be a linear code over a finite chain ring with minimum distance d and let σ be the sum of the Singleton defects of C and $C^{\perp}$. Our formula shows that the knowledge of $\sigma + d - 1$ elements of the weight distribution of C is enough to compute the full weight distribution of C and $C^{\perp}$. Thus, one can prove directly the weight distributions of some families of codes, such as free MDS codes: in this case the knowledge of the length n and the minimum distance d are sufficient to derive the full weight distribution of the code.

Keywords: Codes over rings; Weight distributions, Finite chain rings

MSC(2010): 94B05, 13F10

References

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Near–MDS codes and caps

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Joint work with: Antonio Cossidente, Giuseppe Marino and Francesco Pavese

Abstract

Let q be a power of a prime and $\mathrm{PG}(k-1,q)$ the projective space of dimension $k-1$ over $\mathbb{F}_q$. We call *n-cap* a point set of size n such that no three of them are collinear; it is complete if it is not contained in any $(n+1)$-cap. If we take the matrix whose columns are the representative of the points of an n-cap, we get the parity-check matrix of a linear code over $\mathbb{F}_q$. Moreover, if $n > k$, complete n-caps of $\mathrm{PG}(k-1,q)$ are essentially equivalent to non-extendable linear $[n, n-k, 4]_q$ codes with covering radius $\rho = 2$.

For any $[n,k,d]_q$ linear code, the *Singleton defect* is $D := n-k+1-d$. We call *near-MDS* a code such that both itself and its dual have $D = 1$ and this is equivalent to say that the columns of a generator matrix form a set of points in $\mathrm{PG}(k-1,q)$, $k \geq 3$ (called NMDS-set) with the following three properties: every $k-1$ points generate a hyperplane, there are k points belonging to the same hyperplane and every $k+1$ points generate the whole $\mathrm{PG}(k-1,q)$. An NMDS-set is *complete* if it is maximal with respect to inclusion.

In this talk, based on the paper [1], we will examine NMDS-sets of dimension 4 and caps in $\mathrm{PG}(4,q)$. In particular we will see: a class of NMDS-sets of $\mathrm{PG}(3,q)$, $q = 2^{2h+1}$, $h \geq 1$, obtained intersecting an elliptic quadric and a Suzuki–Tits ovoid of $W(3,q)$ (size: $q + \sqrt{2q} + 1$), two classes of complete caps of $\mathrm{PG}(4,q)$, derived by the previous result (size: $2q^2 - q \pm \sqrt{2q} + 2$) and the possible sizes of an NMDS-set containing a twisted cubic of $\mathrm{PG}(3,q)$.

Keywords: nearMDS codes; caps

MSC(2010): 51E22, 94B05, 51E20

References

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Achromatic Dominating Sets

Nancy E. Clarke

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Joint work with: Ruth Haas

Abstract

Given a k-colouring of (the vertices of) a graph G, a dominating set D of G is said to be an achromatic (or rainbow) dominating set if every vertex of D has a different colour. Our parameter of interest is the achromatic dominating number $\rho(G)$, defined to be the minimum number of colours such that, for any $\rho(G)$-colouring of G, there exists an achromatic dominating set. In this talk, we present a variety of results including exact values of our parameter for several classes of graphs, as well as more general bounds. In particular, we consider graphs of diameter 2 and lexicographic products.

Keywords: Graphs, dominating sets, achromatic, diameter, products

MSC(2010): 05C15, 05C69, 05C76

Egalitarian Popularity Labellings for Steiner Systems

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Joint work with: Dylan Lusi

Abstract

Steiner systems and their duals are widely used for data layout in distributed storage systems. However, the specific assignment of data items to storage units often ignores the long-term popularity of the items. In addressing popularity, a general problem is:

> Order the blocks of a design, computing the *point sum* of an element as the sum of the indices of blocks containing that element. The *point difference sum* is the difference between the largest and smallest point sums. Find a block ordering that minimizes the point difference sum.

A block ordering is *egalitarian* when its point difference sum is zero. In this talk, we explore bounds on the difference sums for Steiner systems $S(t, k, v)$ in general. We outline a construction for egalitarian orderings of certain $S(2, k, v)$ designs that are resolvable and 1-rotational, and suggest methods to extend this construction.

Keywords: egalitarian labelling; Steiner system; 1-rotational design

MSC(2010): 05B05, 05B25

On the number of non-isomorphic (simple) k-gonal biembeddings of complete multipartite graphs

Simone Costa

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Joint work with: Anita Pasotti

MSC(2010): 05C10, 05C15, 05B20, 54C25

Abstract

In this talk we provide exponential lower bounds on the number of non-isomorphic k-gonal biembeddings of the complete multipartite graph into orientable surfaces. For this purpose, we use the concept, introduced by Archdeacon in 2015, of Heffer array and its relations with graph embeddings. In particular we show that, under certain hypotheses, from a single Heffter array, we can obtain an exponential number of distinct graph embeddings. Exploiting this idea starting from the arrays constructed by Cavenagh, Donovan and Yazıcı in 2020, we obtain that, for infinitely many values of k and v, there are at least $k^{\frac{k}{2}+o(k)} \cdot 2^{v \cdot \frac{H(1/4)}{(2k)^2}+o(v)}$ non-isomorphic k-gonal biembeddings of K_v, where $H(\cdot)$ is the binary entropy function. Moreover about the embeddings of $K_{\frac{v}{t} \times t}$, for $t \in \{1, 2, k\}$, we provide a construction of $2^{v \cdot \frac{H(1/4)}{2k(k-1)}+o(v,k)}$ non-isomorphic k-gonal biembeddings whenever k is odd and v belongs to a wide infinite family of values.

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From Kirby diagrams to edge-colored graphs representing PL 4-manifolds

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Joint work with: Maria Rita Casali

Abstract

Regular edge-colored graphs have proved to be, in many ways, an useful combinatorial tool to encode triangulations and thus represent compact PL manifolds of any dimension.

On the other hand, in dimension 4, a classical representation method for compact PL manifolds is given by *Kirby diagrams*, a Kirby diagram being a link L in the 3-sphere equipped with a vector d of integers associated to some of its components.

We present an algorithm which, given a Kirby diagram (L, d) of a compact PL 4-manifold $M^4(L, d)$, produces an edge-colored graph representing $M^4(L, d)$ and directly "drawn" over a planar diagram of L. Furthermore, the combinatorial structure of the resulting graph allows to obtain upper bounds for the value of some graph-defined invariants of $M^4(L, d)$.

If, in particular, we apply our method to Kirby diagrams of exotic 4-manifolds, it can provide us with a large number of examples of explicit triangulations of such manifolds, which can be otherwise rarely found in literature.

Finally, in case $M^4(L, d)$ has empty or connected boundary, we show how the graphs produced by the algorithm induce a particular kind of decompositions of the manifold, called *trisections*, which were introduced by Gay and Kirby in 2016 and are still intensively studied.

Keywords: Kirby diagram; PL 4-manifold; edge-colored graph; trisection

MSC(2010): 57K40, 57M15, 57Q15

Partial difference sets in nonabelian groups

James A Davis

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Joint work with: Andrew Brady

Abstract

Partial difference sets (PDSs) in nonabelian groups are relatively rare. In this talk, we describe how recent computer searches for PDSs in groups of order 64 have revealed that linking systems of difference sets can be used to construct PDSs in many nonabelian groups. We indicate how this can be used to construct nonabelian PDSs in larger groups.

Keywords: Partial difference sets, Linking systems

MSC(2010): 05B20

An Erdős-Ko-Rado problem on flags of finite spherical buildings

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Joint work with: Sam Mattheus and Klaus Metsch

Abstract

The following theorem is one of the earliest Erdős-Ko-Rado (EKR) theorems in finite geometry, see e.g. [2]. Let $n \in \mathbb{N}$ and $n \geq 2k$. Let $\mathcal{F}$ be a family of k-dimensional subspaces of $V(n,q)$, pairwise intersecting non-trivially. Then $|\mathcal{F}| \leq \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q$. In case of equality and $n > 2k$, $\mathcal{F}$ is the set of k-dimensional subspaces that contain a fixed 1-dimensional subspace.

In this talk we will discuss an EKR problem on flags in finite spherical buildings. To state the EKR problem, "non-intersection of subspaces" will be replaced by "opposite flags". Hence we are looking for a set of flags mutually non-opposite, i.e. a coclique in the oppositeness graph. We explain how the upper bound is found by applying Hoffman's ratio bound (also known as the Delsarte-Hoffman coclique bound). This requires the application of an algorithm found in [1], and it will be explained how it can be applied in classical geometries such as finite projective spaces and finite classical polar spaces. Also attention will be given to remaining open problems and ongoing work.

Keywords: Association scheme, Erdős-Ko-Rado problem, flags of finite projective spaces

MSC(2010): 05B25, 05E30, 51E20, 51E24

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The weight distributions of linear sets in PG$(1, q^5)$

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Joint work with: Geertrui Van de Voorde

Abstract

A set S is said to be an $\mathbb{F}_q$-*linear set* of *rank* k in $\mathrm{PG}(r-1, q^t) = \mathrm{PG}(W)$ if $S = L_U$, with

$$L_U = \{\langle v\rangle_{q^t} \mid v \in U \setminus \{0\}\},$$

where U is a k-dimensional $\mathbb{F}_q$-vector subspace of $W = \mathbb{F}_{q^t}^r$ and $\langle v\rangle_{q^t}$ denotes the projective point determined by the vector v. Linear sets are central objects in modern day projective geometry, with applications in coding theory, algebra and projective geometry itself.

For a point P in L_U, the t-dimensional $\mathbb{F}_q$-vector space defining P intersects U in an i-dimensional $\mathbb{F}_q$-vector space for some $i > 0$. The integer i is called the *weight* of the point P (see [3]). This allows to define the *weight distribution* of a linear set. For linear sets of rank 1, 2 and 3, there are 1, 2 and 3 admissible weight distributions, respectively. The admissible weight distributions of linear sets of rank 4 were described in [2]. In this talk, based on [1], we will discuss the possible weight distributions of $\mathbb{F}_q$-linear sets of PG$(1, q^5)$.

MSC(2010): 51E20

Keywords: Linear set; weight distribution; club

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On Sequences in Cyclic Groups with Distinct Partial Sums

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Joint work with: S. Costa, M. A. Ollis and S. Rovner–Frydman

Abstract

Let S be a subset of $\mathbb{Z}_n \setminus \{0\}$ of size k. Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_k)$ be an ordering of the elements of S and define its partial sums $\boldsymbol{y} = (y_0, y_1, \ldots, y_k)$ by $y_0 = 0$ and $y_i = x_1 + \cdots + x_i$ for $i > 0$. Alspach [1] conjectures that every subset S whose sum is nonzero has an ordering with distinct partial sums. The successful/partial resolution of this conjecture has implications in the study of graph decompositions and embeddings and in the construction of non-zero sum Heffter arrays. Inspired by previous works (see [3]), we translate the problem into one of finding monomials with non-zero coefficients in particular polynomials over $\mathbb{Z}_p$, where p is a prime divisor of n, using Alon's Combinatorial Nullstellensatz. The approach can be used in conjunction with a computational approach in cases where $n = pt$ with p prime and t and k small. In [2] we proved the conjecture for $k = 10, 11$ and $t = 2, 3, 4, 5$ and for $k = 12$ and $t = 1, 2, 3, 4$, and in addition when $S \subseteq \mathbb{Z}_m \times H \setminus \{0_{\mathbb{Z}_m \times H}\}$ and $k \leq 12$, where m has all prime factors greater than $k!/2$ and $H \in \{\{0\}, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_4\}$.

MSC(2010): 05C25, 20K15

Keywords: Alspach's Conjecture, partial sums, polynomial method

References

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Higgledy-piggledy sets in projective spaces

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Abstract

In this talk, we focus on *higgledy-piggledy sets* of k-subspaces in $\mathrm{PG}(N, q)$, i.e. sets of projective subspaces that are 'well-spread-out'. More precisely, the set of intersection points of these k-subspaces with any $(N-k)$-subspace κ of $\mathrm{PG}(N, q)$ spans κ itself. In other words, the set of points in the union of these k-subspaces forms a *strong blocking set* w.r.t. $(N-k)$-subspaces. Naturally, one would like to find a higgledy-piggledy set consisting of a small number of k-subspaces.

Although these combinatorial sets of subspaces are sporadically mentioned in older works, only since 2014 researchers have started to investigate these sets as a main point of interest. This talk aims to give its audience an overview of known results concerning higgledy-piggledy sets (lower bounds, existence results, construction methods...) and their applications to coding and graph theory, as well as share some new results and interesting open problems.

Keywords: Cutting blocking sets; Higgledy-piggledy sets; Minimal codes; Projective spaces; Strong blocking sets

MSC(2010): 05B25, 94B05, 51E20, 51E21

References

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The Segre variety $\mathcal{S}_{2,2}(\mathbb{K})$ in a geometry of type $\mathsf{E}_{6,1}(\mathbb{K})$

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Abstract

The Sege variety $\mathcal{S}_{2,2}(\mathbb{K})$, or more abstractly even any direct product of two projective planes over $\mathbb{K}$, fully embeds in four different ways in an exceptional geometry of type $\mathsf{E}_{6,1}(\mathbb{K})$. I will give some properties of both geometries, and focus on their full embeddings in each other. The motivation for this lies in the full embedding of the long root geometry $\mathsf{E}_{6,2}(\mathbb{K})$ in $\mathsf{E}_{8,8}(\mathbb{K})$. With our results, we can prove that there is, up to projectivity, a unique such embedding, arising in a beautiful geometric way.

Keywords: Exceptional geometries, Segre variety, embeddings

MSC(2010): 51E24, 51A45

New examples of Cameron-Liebler sets in hyperbolic quadrics

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Joint work with: M. De Boeck and M. Rodgers

Abstract

In 1982, Cameron and Liebler introduced specific line classes in $\mathrm{PG}(3,q)$ when investigating the orbits of the subgroups of the collineation group of $\mathrm{PG}(3,q)$. They found that these Cameron-Liebler sets can be defined in many equivalent ways; some combinatorial, geometrical or algebraic in nature.

A Cameron-Liebler line set $\mathcal{L}$ in $\mathrm{PG}(3,q)$ is a set of lines, such that every line spread in $\mathrm{PG}(3,q)$ has the same number of lines in common with $\mathcal{L}$.

The examination of these Cameron-Liebler line sets in $\mathrm{PG}(3,q)$ started the motivation for defining and investigating Cameron-Liebler sets in other contexts, including the context of finite classical polar spaces [1, 2].

In this talk I will focus on Cameron-Liebler sets in these finite classical polar spaces. I will present some non-trivial examples of Cameron-Liebler sets of generators in hyperbolic quadrics, which were recently found by M. De Boeck, M. Rodgers and myself.

Keywords: Cameron-Liebler set, Polar spaces, Hyperbolic quadrics

MSC(2010): 51A50, 51E20, 05B25, 05E30

References

[1] M. De Boeck, M. Rodgers, A. Švob. "*Cameron-Liebler sets of generators in finite classical polar spaces*", J. Combin. Theory Ser. A, **167**: 340–388, 2019.

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Intersecting theorems for finite general linear groups

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Joint work with: Kai-Uwe Schmidt

Abstract

A subset Y of the symmetric group $\mathcal{S}_n$ is t-intersecting if $x^{-1}y$ fixes t elements in $[n]$ for all $x, y \in Y$. Deza and Frankl conjectured [1] and Ellis, Friedgut, and Pilpel proved [2] that the size of a t-intersecting set in $\mathcal{S}_n$ is at most $(n-t)!$ for n sufficiently large compared to t. Moreover equality holds if and only if Y is a coset of the stabiliser of a t-tuple.

In this talk we discuss a q-analog of this result. We define a subset Y of $\mathrm{GL}(n,q)$ to be t-intersecting if $x^{-1}y$ fixes a t-dimensional subspace of $\mathbb{F}_q^n$ pointwise for all $x, y \in Y$. It is shown that the size of a t-intersecting subset of $\mathrm{GL}(n,q)$ is at most

$$[n-t]_q! \frac{(q-1)^n q^{\binom{n}{2}}}{(q-1)^t q^{\binom{t}{2}}} = (q^n - q^t)(q^n - q^{t+1}) \cdots (q^n - q^{n-1})$$

for n sufficiently large compared to t. Moreover we give a characterisation of the cases for which equality holds.

Keywords: Association scheme, Erdős-Ko-Rado, intersecting, finite general linear groups

MSC(2010): 05D99, 05E30, 20C30

References

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Permutations of zero-sum k-sets

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Joint work with: Marco Pavone

Abstract

We consider the natural partition of the family of all k-subsets of a a vector space P over a Galois field $\mathrm{GF}(p)$, respectively $P^* = P \setminus \{0\},\}$ as the disjoint union, of the families of all k-sets of elements adding up to zero in P, respectively in P^*.

A natural question to ask is: what are the permutations of P (respectively of P^*) that, for a given k, induce permutations of the family of all k-sets of elements adding up to 0? We prove that the only such permutations of P are the invertible linear mappings, if p does not divide k, and the invertible affinities of the affine space P over $\mathrm{GF}(p)$, if p divides k. Also, we prove that the only such permutations of P^* are the invertible linear mappings of P over $\mathrm{GF}(p)$ [1]. When the family B_k^0 of zero-sum k-sets represents the blocks of a 2-design $D = (P, B_k^0)$, the permutations of P that induce permutations of B_k^0 form precisely the automorphism group of the block design D. The same questions may be asked for $D^* = (P^*, B_k^{0,*})$.

Connections to Hamming codes will be outlined [2].

Keywords: Zero-sum sets; Additive designs.

MSC(2010): 05A18, 11P70

References

[1] Falcone, G., Pavone, M. "*Permutations of zero-sumsets in a finite vector space*", Forum Mathematicum, **33** (2) 2021.

[2] Falcone, G., Pavone, M. "*Binary Hamming codes and Boolean designs*", Designs Codes and Cryptography, **89** (6) 2021.

Homology of directed graphs with application to DNA recombination

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Joint work with: Lina Fajardo Gómez, Nataša Jonoska, Masahico Saito

Abstract

A double occurrence word (DOW) is a word in which every symbol appears exactly twice. We consider subwords which occur twice (repeat word) or which occur once along with their reverse (return word). Such subwords generalize square and palindromic factors of DOWs, respectively. In the context of genomics, deletions of repeat and return words on DOWs have been used to study DNA recombination in certain species of ciliates [1]. We model these processes with directed graphs where vertices are DOWs, and an edge from w to w' exists if w' is obtained from w through a repeat/return word deletion. On a directed graph, we consider the cell complex consisting of products of directed simplices and then compute homology groups, which can be used to describe the complexity of these recombination processes.

Keywords: Directed graph, homology, DNA recombination.

MSC(2010): 92B05, 68R15, 55N31

References

[1] J. Burns, D. Kukushkin, X. Chen, L. Landweber, M. Saito, N. Jonoska. "*Recurring Patterns Among Scrambled Genes in the Encrypted Genome of the Ciliate* Oxytricha trifallax", Journal of Theoretical Biology, **410** 171–180, (2016).

Γ-supermagic labelings of $C_m \square C_n$

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Joint work with: P. Paananen and L. Sorensen

Abstract

There is a close connection between Abelian groups and Cartesian products of cycles, since every Cartesian product $C_{n_1} \square C_{n_2} \square \cdots \square C_{n_t}$ can be viewed as the Cayley graph of the group $Z_{n_1} \oplus Z_{n_2} \oplus \cdots \oplus Z_{n_t}$ with generating set $S = \{(1, 0, \ldots, 0), (0, 1, 0, \ldots, 0), \ldots, (0, 0, \ldots, 1)\}$.

A graph $G = (V, E)$ is Γ-*supermagic* if there exists a bijection f from E to a group Γ of order $|E|$ (called a Γ-*supermagic labeling*) such that the *weight* $w(x)$ of each vertex x, defined as the sum of labels of all edges incident with x, is equal to the same *magic element* μ. That is, there exists $\mu \in \Gamma$ such that for all $x \in V$,

$$w(x) = \sum_{xy \in E} f(xy) = \mu.$$

It was proved by DF, McKeown, McKeown, and McKeown ([1], [2]) that a Z_{2mn}-supermagic labeling of $C_m \square C_n$ exists for all $m, n \geq 3$. We prove that when $m \equiv n \pmod 2$, then $C_m \square C_n$ allows a Γ-supermagic labeling by any Abelian group Γ of order $2mn$. We also present some preliminary results on labelings of $C_m \square C_n$ by non-Abelian groups, namely dihedral groups D_{mn}.

Keywords: Γ-supermagic labeling, Γ-supermagic graph, vertex-magic edge Γ-labeling

MSC(2010): 05C78

References

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[2] D. Froncek, M. McKeown, "*Note on diagonal construction of Z_{2nm}-supermagic labeling of $C_n \square C_m$*", AKCE Int. J. Graphs Comb., **17**(3) 952–954, 2020.

Steiner loops of affine and projective type

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Joint work with: Giovanni Falcone, Ágota Figula

Abstract

In this talk I will distinguish Steiner loops of *affine* and of *projective* type, showing common features and differences between them. After some definitions and basic properties, I will focus on the correspondence between normal subloops and quotient loops, and Steiner subsystems and induced Steiner quotient systems, respectively. Indeed, if $\mathcal{L}_\mathcal{S}$ is a Steiner loop of affine/projective type corresponding to a STS $\mathcal{S}$, there is a bijection between the subloops of $\mathcal{L}_\mathcal{S}$ and the subsystems of $\mathcal{S}$ (containing the identity element for the affine case). If $\mathcal{L}_\mathcal{S}$ has a normal subloop $\mathcal{L}_\mathcal{N}$, any coset modulo $\mathcal{L}_\mathcal{N}$ corresponds to a STS (containing $\mathcal{N}$ for the projective case), and also the quotient loop $\mathcal{L}_\mathcal{S}/\mathcal{L}_\mathcal{N}$ is a Steiner loop of affine/projective type. I will report some new results about projective Steiner loops that are extension of a normal subloop $\mathcal{L}_\mathcal{N}$ by a quotient loop $\mathcal{L}_\mathcal{Q}$, comparing this construction with the affine case.

Keywords: Steiner triple systems; loops.

MSC(2010): 51E10, 20N05

References

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Weil polynomials of abelian varieties over finite fields with many rational points

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Joint work with: Elena Berardini

Abstract

In this work we present a connection between abelian varieties over finite fields with many rational points, their group structure, and algebraic integers with minimal trace. This allow us to have some information about the Weil polynomial of these varieties, and to show that their group of rational points are cyclic outside a explicitly given set of primes. This talk is based on a recently published paper [1].

Keywords: Abelian varieties over finite fields; Weil polynomials; groups of rational points; cyclic groups

MSC(2010): 11G10, 14G05, 14G15, 14K02

References

[1] Elena Berardini, Alejandro J. Giangreco-Maidana. "*Weil polynomials of abelian varieties over finite fields with many rational points*", International Journal of Number Theory, **0(0):1–13, 2022.**

Graphs with few hamiltonian cycles

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Joint work with: Barbara Meersman and Carol T. Zamfirescu

Abstract

We will present an algorithm for the exhaustive generation of all non-isomorphic graphs with a given number $k \geq 0$ of hamiltonian cycles, which is especially efficient for small values of k. Our main findings, combining applications of this algorithm and existing algorithms with new theoretical results, revolve around graphs containing exactly one hamiltonian cycle – i.e. uniquely hamiltonian (UH) graphs – or exactly three hamiltonian cycles.

Motivated by a classic result of Smith and recent work of Royle, we show that there exist nearly cubic UH graphs of order n iff $n \geq 18$ is even. This gives the strongest form of a theorem of Entringer and Swart, and sheds light on a question of Fleischner originally settled by Seamone.

We prove equivalent formulations of the conjecture of Bondy and Jackson that every planar UH graph contains two vertices of degree 2, verify it up to order 16, and show that its toric analogue does not hold.

Furthermore, we verify the conjecture of Sheehan that there is no 4-regular UH graph up to order 21 and answer a question of Chia and Thomassen on the number of longest cycles in cyclically 4-edge-connected planar cubic graphs.

Keywords: Hamiltonian cycle; uniquely hamiltonian; Bondy-Jackson conjecture; cubic graph; exhaustive generation

MSC(2010): 05C10, 05C45, 05C85

Absolute points of correlations of PG$(5, q^n)$

Giovanni G. Grimaldi

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Joint work with: Nicola Durante

Abstract

The sets of the absolute points of (possibly degenerate) polarities of a projective space are well known. The sets of the absolute points of (possibly degenerate) correlations, different from polarities, of PG$(2, q^n)$ have been completely determined by B.C. Kestenband in 11 papers from 1990 to 2014, for non-degenerate correlations and by J. D'haeseleer and N. Durante in [3] for degenerate correlations. The sets of the absolute points of degenerate correlations, different from degenerate polarities, of PG$(3, q^n)$ and PG$(4, q^n)$ have been determined respectively in [2] and [1]. In this talk, we consider the five dimensional case and determine the sets of the absolute points of degenerate correlations, different from degenerate polarities, of a projective space PG$(5, q^n)$. As an application we show that some of these sets are related to some ovoids of PG$(5, q^n)$.

Keywords: Sesquilinear forms, correlations.

MSC(2010): 51E20, 05B25

References

[1] J. D'haeseleer, N. Durante. "*On absolute points of correlations in* PG$(2, q^n)$", Electron. J. Combin., **27(2):2–32, 2020.**

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[3] N. Durante, G.G. Grimaldi. "*Absolute points of correlations in* PG$(4, q^n)$", Submitted.

Il mantovano Vittorio Martinetti (1859-1936) and the future of configurations

Harald Gropp

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Abstract

Vittorio Martinetti was born in Scorzalo near Mantova in 1859, at that time still part of the Habsburg monarchy. Since the Combinatorics 2022 takes place in Mantova, a short biography should be part of this talk. He worked on configurations and introduced spatial configurations. After his stay with the universities in Messina and Palermo he came back to the north of Italy and died in Milano in 1936.

A second reason for this talk is that among the participants there are several who contributed to the research on configurations in the last 30 years. These are T. Pisanski, P. Östergård, V. Krčadinac, K. Stokes and our Polish colleagues. I hope very much I did not forget anybody.

It may be useful to look back to the last 40 years of research on configurations and to plan the future years.

In hypergraph language configurations are linear regular uniform hypergraphs. They were born in geometrical language in 1876 as a system of points and lines such that every line contains k points, through every point there are r lines, and through 2 different points there is at most one line.

At the end of the 19th century Martinetti introduced spatial configurations where instead through 2 different points there are at most 2 lines. The focus of this talk will probably be on these spatial or 2-configurations since there is even more to investigate in the future.

So far a short abstract of this talk in Mantova, the northernmost town of ancient Etrurian culture with a kakadu in a church.

Keywords: Configurations

MSC(2010): 05B30

Minimal codes, strong blocking sets and higgledy-piggledy lines

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Joint work with: Zoltán Lóránt Nagy

Abstract

A codeword v of a linear code $\mathcal{C}$ is *minimal* if the support of v does not contain the support of any codeword other than its scalar multiples. The linear code $\mathcal{C}$ is a *minimal code* if all of its codewords are minimal.

The geometrical interpretaiton of minimal codes as point sets of a projective space are called strong (or cutting) blocking sets. A point set B of $\mathrm{PG}(n,q)$ is a *strong blocking set* if for every hyperplane H of $\mathrm{PG}(n,q)$, $H \cap B$ generates H; that is, $H \cap B$ contains n points in general position. Note that in $\mathrm{PG}(2,q)$, strong blocking sets are the same as double blocking sets.

A major problem regarding strong blocking sets is to find small examples. Formerly known constructions were either large (quadratic in n or superlinear in q) or required q to be large compared to n. In the talk, we review the idea of constructing strong blocking sets based on so-called higgledy-piggledy lines. Furthermore, we present a simple construction formed by a random set of higgledy-piggledy lines which works for all q and n, and whose size is linear in both n and q. Although the topic originates from coding theory, our focus will be on the geometrical point of view.

Keywords: Strong blocking set; cutting blocking set; minimal code; higgledy-piggledy lines.

MSC(2010): 94B05, 05B25, 51E21

References

[1] Tamás Héger and Zoltán Lóránt Nagy. "*Short Minimal Codes and Covering Codes via Strong Blocking Sets in Projective Spaces*", IEEE Transactions on Information Theory, **68(2):881–890, 2022.**

Finding cycles in supersingular isogeny graphs

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Joint work with: Jenny G. Fuselier, Mark Kozek, Travis Morrison and Changningphaabi Namoijam.

Abstract

Supersingular isogeny graphs are graphs whose vertices correspond to (isomorphism classes of) supersingular elliptic curves over finite fields and whose edges represent (equivalence classes of) isogenies between elliptic curves. A cycle in such graphs corresponds to an isogeny from a supersingular curve E (of the cycle) to itself, i.e. to an endomorphism of E. Computing non-trivial endormorphisms of a supersinguar elliptic curve , i.e. finding cycles in a supersingular isogeny graph, is a computationally difficult problem which underlies the hard problem of computing the endomorphism ring of a supersingular elliptic curve over a finite field.

A motivation for this computational problem comes from cryptography, and more precisely from a young subfield of post-quantum cryptography called isogeny-based cryptography. The security of an isogeny-based cryptosystem is based on the mathematical problem of computing an isogeny between two elliptic curves E and E' and it has been shown that, in the supersingular case, this problem can be reduced to the computation of the endomorphism rings of E and E'.

In this talk, after reviewing the mathematical and cryptographic context, we will present an improved algorithm for computing the endomorphism ring of a supersingular elliptic curve over a finite field.

Keywords: Supersingular elliptic curves; endomorphism ring; isogeny-based cryptography etc.

MSC(2010): 11G05, 11T71

Substructures in long root geometries

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Joint work with: Hendrik Van Maldeghem

Abstract

Spherical buildings can be studied by looking at them from an incidence geometrical perspective: to any spherical building one can attach the so called long root geometry. For example, for a building $A_n(K)$, these are the point-hyperplane flags of the projective space $PG(n, K)$. For a building $D_n(K)$ this is the linegrassmannian of the thin polar space of rank n.

The points of a long root geometry of a building of type X_n coincide locally (i.e. in an appartment) with the long roots of a root system ϕ of type X_n. By studying certain subsets this root system ϕ, one can uncover interesting substructures of the long root geometry (and of other geometries attached to X_n). In this talk, we discuss this general phenomenon and zoom in on certain examples, in particular those involving exceptional buildings E_6, E_7, E_8.

Keywords: Long root geometries, Projective spaces, Polar spaces, Root systems, Exceptional groups, Spherical buildings

MSC(2010): 05B99, 05E20

Linear sets without points of weight one

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and
University of Canterbury (New Zealand) — School of Mathematics and Statistics

Joint work with: Geertrui Van de Voorde

Abstract

Linear sets in finite projective spaces can be thought as a generalization of subgeometries. But unlike subgeometries, different linear sets with the same rank and size can still exhibit widely different behaviour. Because of their use in many applications, *scattered* linear sets (those of maximum size for a fixed rank) have received more attention than other linear sets. In this talk we will focus on the other side of the weight spectrum.

De Beule and Van de Voorde [3] provided a lower bound on the size of an $\mathbb{F}_q$-linear set of rank k with at least one point of weight one. In this talk, we will explain how going below this limit with all points of weight more than one leads to interesting results.

Keywords: Linear set

MSC(2010): 51E20

References

[1] J. De Beule, G. Van de Voorde. "*The minimum size of a linear set*", J. Combin. Theory, **A(164):109–124, 2019.**

Internal and External Partial Difference Families

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Joint work with: Sophie Huczynska

Abstract

A Disjoint Difference Family (DDF) is a combinatorial structure formed from a collection of disjoint subsets of a group G, in which each group element occurs precisely lambda times as a difference between two elements of the same subset. An External Difference Family (EDF) is similarly formed by disjoint subsets of G, with each element of G occurring exactly lambda times as a difference between elements of disjoint subsets. Both combinatorial structures have been widely studied and have applications to cryptography.

In previous research by Chang and Ding, it was observed that a collection of subsets partitioning G forms a DDF if and only if it also forms an EDF. Extending upon this idea, we began looking at partitions of $G\backslash\{0\}$, Difference Sets (DSs) and Partial Difference Sets (PDSs). Our investigations gave rise to two new combinatorial structures, namely a Disjoint Partial Difference Family and External Partial Difference Family.

In this talk, I will introduce these structures, present constructions for DPDFs and EPDFs and detail some applications of these structures.

Keywords: Internal and External Partial Difference Families, Cyclotomy, Difference Families

MSC(2010): 05B10, 11T22, 94A60

References

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The line graph of Lollipop and Pan graphs

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Joint work with: Vito Napolitano

Abstract

Line graphs are a rich and well-studied class of graphs. In this talk, I will present some results on the line graph of Pan and Lollipop graph. In particular, the domination, the chromatic and the independence numbers of the line graphs of Lollipop and Pan graphs will be given. Also, I will discuss the relation between the chromatic number and domination number of Pan graph with the line graphs of Pan graph.

Keywords: Line graph; Pan graph; Lollipop graph; Independence number; Domination number; Chromatic number.

MSC(2010): 05C45, 05C40, 05C90

References

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Girth-(bi)regular graphs and finite geometries

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Joint work with: Š. Miklavič and T. Szőnyi

Abstract

In this talk we consider girth-regular and girth-biregular graphs. Let Γ denote a simple, connected, finite graph. For an edge e of Γ let $n(e)$ denote the number of girth cycles containing e. For a vertex v of Γ let $\{e_1, e_2, \ldots, e_k\}$ be the set of edges incident to v ordered such that $n(e_1) \leq n(e_2) \leq \cdots \leq n(e_k)$. Then $(n(e_1), n(e_2), \ldots, n(e_k))$ is called the *signature* of v. The graph Γ is said to be *girth-(bi)regular* if (it is bipartite, and) all of its vertices (belonging to the same bipartition) have the same signature.

We show that girth-(bi)regular graphs are related to (biregular) cages, finite projective and affine spaces and generalized polygons. We also present results in the spirit of stability theorems: we give upper bounds on $n(e_k) \leq M$ and show that in the case when $n(e_k) = M - \epsilon$ for some non-negative integer ϵ, then $\epsilon = 0$.

Keywords: girth cycle, girth-regular graph, generalized polygons

MSC(2010): 05C38, 51E12

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Best match graphs and generalizations

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Joint work with: M. Hellmuth, D. Schaller, and P. F. Stadler

Abstract

Recent investigations in mathematical phylogenetics [1, 2] have focused on quasi-best match graphs (qBMGs) which is a hereditary family of properly vertex colored digraphs. qBMGs generalize best match graphs (BMGs) which model the most closely related genes in a phylogentic tree. In this talk, we will see some properties of BMGs, including the hierarchy-like structure of the out-neighborhoods, bipartition, bi-transitivity, acyclicity of underlying orientations, and characterization in terms of forbidden induced subgraphs. Some of these properties also hold for qBMGs; however, whether this family of graphs also have properties that well fit in structural graph theory has not been completely investigated yet.

Keywords: Colored directed graphs, hierarchies, phylogentic combinatorics

MSC(2010): 46N60, 05C20, 03D55

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(k, n)-arcs and algebraic curves

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Joint work with: G.P. Nagy and T. Szőnyi

MSC(2010): 05B25, 51E21, 11R45, 51E20

Abstract

In $PG(2, q)$, a (k, n)-*arc* is a set $\mathcal{K}$ of k points such that some line meets $\mathcal{K}$ in n points but no line meets $\mathcal{K}$ in more than n points. A (k, n)-arc $\mathcal{K}$ is *complete* if each point in $PG(2, q)$ lies on a line meeting $\mathcal{K}$ in n points. (k, n)-arcs are truly combinatorial objects and have been investigated since the pioneering work of B. Segre and A. Barlotti. The classical examples of complete (k, n)-arcs in $PG(2, q)$ are the non-degenerate conic for odd q, $k = q + 1, n = 2$, and the Hermitian unital consisting of all points of the non-degenerate Hermitian curve for $k = q\sqrt{q} + 1, n = \sqrt{q} + 1$. Our objective is to obtain further plane algebraic curves $\mathcal{C}$ whose points in $PG(2, q)$ form a complete (k, n)-arc. In our investigation we adopt the approach used by D. Bartoli and G. Micheli, depending on both combinatorics and geometric methods from theory of algebraic curves over finite fields. The essential idea is to express the condition that any point $P \in PG(2, q)$ is incident with a line which meets $\mathcal{C}$ in n pairwise distinct points, in terms of the Galois closure of the algebraic extension $F|F_P$ where F is the function field of $\mathcal{C}$ and F_P is the rational subfield of F arising from the projection of $\mathcal{C}$ from P. The most favorable situation in this context occurs when the Galois group of the Galois closure of $F|F_P$ is the symmetric, the alternating group or the 2-dimensional projective linear group acting naturally on the roots of the polynomial associated with $F|F_P$. In fact, in such cases classical density theorems, like Chebotarev theorem, work well and may provide a proof for the completeness of the (k, n)-arc.

Keywords: (k, n)-arc, Galois closure, density theorem

Polarity transformations of semipartial geometries

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Joint work with: M. Abreu, M. Funk, and D. Labbate

Abstract

A family of semipartial geometries $LP(n,q)$ was constructed by I. Debroey and J. A. Thas [2] from the lines and planes of a projective space $PG(n,q)$, $n \geq 3$. We will describe transformations of $LP(4,q)$ using polarities of a hyperplane in $PG(4,q)$ or the quotient geometry $PG(4,q)/P_0$ with respect to a point P_0. The resulting incidence geometries are partial linear spaces with constant point and line degrees. They are not semipartial geometries, but the associated point and line graphs are strongly regular. The transformations were discovered in the paper [1].

Keywords: semipartial geometry; polarity; strongly regular graph

MSC(2010): 05B30, 05E30

References

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The graph of 4-ary simplex codes of dimension 2

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Joint work with: Mark Pankov

Abstract

Linear $[n,k]_q$ codes are k-dimensional subspaces of an n-dimensional vector space over the field with q-elements. Projective codes are codes whose every two columns of their generating matrices are non-proportional. Two distinct linear codes are adjacent vertices of the Grassmann graph if they have the maximal possible number of common codewords, i.e. the dimension of their intersection is $k-1$ dimensional. The restriction of the Grassmann graph to projective codes is considered in [1]. Projective $[n,k]_q$ codes exist if $\frac{q^k-1}{q-1} \geq n$ in the case when this is an equality, projective codes are called q-ary simplex codes of dimension k. In[2] we focus on the subgraph of 4-ary simplex codes of dimension 2, it is a 25-regular graph on 162 vertices. We give a complete description of the distance relation on this graph. This is a connected graph of diameter 3. (the diameter of the corresponding Grassmann graph is 2). For every vertex we determine the sets of all vertices at distance $1,2,3$ from it, and give a description of the automorphism group of this graph. This graph is not distance or flag transitive, but the automorphism group acts transitively on pairs of vectors at distance 3.

MSC(2010): 51E22, 94B27

Keywords: Projective codes, Simplex codes.

References

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Binary Non-linear Codes with Two Distances

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Joint work with: Assia Rousseva and Konstantin Vorobev

Abstract

In this paper, we consider the problem of determining the exact value of $A_2(n, \{d_1, d_2\})$ defined as the maximal cardinality of a binary code of length n with two possible distances d_1 and d_2. We prove that if $d_2 > 2d_1$, it holds $A_2(n, \{d_1, d_2\}) \leq n+1$. A similar bound is proved for codes with $d_1 \not\equiv d_2 \pmod 2$:

$$A_2(n, \{d_1, d_2\}) \leq \begin{cases} n+1 & \text{for } d_1 \text{ even}, \\ n+2 & \text{for } d_1 \text{ odd}. \end{cases}$$

Furthermore, we settle two conjectures left open in [1] that imply the following exact values:

$$A_2(n, \{2, d\}) = \begin{cases} \binom{n}{2} + 1 & \text{for } d = 4 \text{ and } n \geq 6, \\ n & \text{for } 4 < d < n-1, \\ n+1 & \text{for } d = n-1, \end{cases}$$

We present some other combinatorial constructions that improve on the lower bounds on $A_2(n, \{d_1, d_2\})$ known so far. Finally, we prove the general upper bound

$$A_2(n, \{d_1, d_2\}) \leq \frac{(n+1)(n+2)}{2}.$$

Keywords: two weight codes; codes with two distances; codes of almost constant weight; quasisymmetric designs

MSC(2010): 05A05, 05A20, 94B25, 94B65

References

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Quasi-Hermitian varieties in $PG(3, q^2)$

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Joint work with: Michel Lavrauw and Francesco Pavese

Abstract

Quasi-Hermitian varieties in $PG(n, q^2)$ have been introduced by De Winter and Schillewaert [2] as combinatorial generalization of (non-degenerate) Hermitian varieties, being sets of points with the same intersection numbers with hyperplanes as a non-degenerate Hermitian variety. In particular quasi-Hermitian varieties are two-character sets, and correspond to strongly regular graphs and two weight codes, [1]. In this talk we present a new construction of such objects in the projective space $PG(3, q^2)$, gluing point-orbits of the group $PSL(2, q^2)$ fixing a rational curve on an Hermitian surface. We also discuss the isomorphism problem with previous constructions, showing that it is actually a new family.

Keywords: Quasi-Hermitian varieties; two character sets; strongly regular graphs

MSC(2010): 05E30, 05B25, 51E30

References

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$(d, \boldsymbol{\sigma})$-Veronese variety and (almost) MDS codes

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Joint work with: N. Durante and V. Pepe

Abstract

Let $\mathbb{K}$ be a field, $A = \mathrm{Aut}(\mathbb{K})$ be the automorphism group of $\mathbb{K}$ and $\boldsymbol{\sigma} = (\sigma_0, \dots, \sigma_{d-1}) \in A^d, d \geq 1$. In this talk the following generalization of the Veronese map

$$\nu_{d,\boldsymbol{\sigma}} : \langle v \rangle \in \mathrm{PG}(n-1, \mathbb{K}) \longrightarrow \langle v^{\sigma_0} \otimes v^{\sigma_1} \otimes \cdots \otimes v^{\sigma_{d-1}} \rangle \in \mathrm{PG}(n^d-1, \mathbb{K}),$$

is introduced and some properties of its image, called here $(d, \boldsymbol{\sigma})$-*Veronese variety* $\mathcal{V}_{d,\boldsymbol{\sigma}}$, are analyzed. In particular, we focus on the case of a finite field $\mathbb{K} = \mathbb{F}_{q^t}$, where $q = p^e$ and p is a prime number. For $d = t$, σ a generator of $\mathrm{Gal}(\mathbb{F}_{q^t}|\mathbb{F}_q)$ and $\boldsymbol{\sigma} = (1, \sigma, \sigma^2, \dots, \sigma^{t-1})$, the $(t, \boldsymbol{\sigma})$-Veronese variety $\mathcal{V}_{t,\boldsymbol{\sigma}}$ was first introduced in [2]. Here, we will show that $\mathcal{V}_{d,\boldsymbol{\sigma}}$ is the Grassmann embedding of a normal rational scroll and any $d+1$ points of it are linearly independent. We give a characterization of $d+2$ linearly dependent points of $\mathcal{V}_{d,\boldsymbol{\sigma}}$ and for some choices of parameters, we obtain that $\mathcal{V}_{p,\boldsymbol{\sigma}}$ is the normal rational curve, the Segre's arc of $\mathrm{PG}(3, q^t)$ for $p = 2$ and, for $p = 3$, it is a $|\mathcal{V}_{p,\boldsymbol{\sigma}}|$-track of $\mathrm{PG}(5, q^t)$. Finally, we investigate the link between such points set and a linear code $\mathcal{C}_{d,\boldsymbol{\sigma}}$ that can be associated to the variety, obtaining examples of MDS and almost MDS codes.

Keywords: Veronesean, Grassmannian, normal rational scroll, almost MDS

MSC(2010): 94B05, 94B27, 15A69, 51E20

References

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On the structure of cube tilings

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Abstract

A family $[0,1)^d + T, T \subseteq \mathbb{R}^d$, of pairwise disjoint cubes such that the union of them is the whole space $\mathbb{R}^d$ is called a cube tiling of $\mathbb{R}^d$. A family of cubes $[0,1)^d + S$, $s \subseteq \mathbb{R}^d$, is said to be an l-column, $1 \leq l < d$, if there is $i \in \{1,\ldots,d\}$ and $\alpha \in \mathbb{R}$, and there are $i_1,\ldots,i_l \in \{1,\ldots,d\}$ and $\alpha_{i_1},\ldots,\alpha_{1_l} \in \mathbb{R}$ such that

$$S = \{s = (s_1,\ldots,s_d) \in \mathbb{R}^d \colon s_i = \alpha, s_{i_1} = \alpha_{i_1},\ldots,s_{i_l} = \alpha_{i_l}\}.$$

Every $(d-1)$-column in $\mathbb{R}^d$ is called a column.

Let $T \subseteq \mathbb{R}^d$ be such that $[0,1)^d + T$ is a cube tiling of $\mathbb{R}^d$ and let $W \subseteq T$. Then the family of cubes $[0,1)^d + W$ is called a cylinder, if there is $i \in \{1,\ldots,d\}$ and $\alpha \in \mathbb{R}$ such that

$$W = \{t = (t_1,\ldots,t_d) \in T \colon t_i \in \alpha + \mathbb{Z}\}.$$

The problem investigated in [1, 2, 3] of the existence of an l-column in a cylinder in a cube tiling of $\mathbb{R}^d$, due to Keller's conjecture (in every cube tiling of $\mathbb{R}^d$ there is a column), which is still open for $d = 7$, will be presented.

MSC(2010): 05B45, 52C22

Keywords: cube tiling; Keller's conjecture; column; cylinder

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Berge's conjecture for cubic graphs with small colouring defect

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Joint work with: Ján Karabáš, Roman Nedela, Martin Škoviera

Abstract

A long-standing conjecture of Berge suggests that every bridgeless cubic graph can be expressed as a union of at most five perfect matchings. This conjecture trivially holds for 3-edge-colourable cubic graphs, but remains widely open for graphs that are not 3-edge-colourable. In this talk we show that Berge's conjecture is true for cubic graphs of colouring defect 3. Colouring defect, or simply defect for short, is a measure of uncolourability of cubic graphs defined as the minimum number of edges left uncovered by any collection of three perfect matchings. While 3-edge-colourable graphs have defect 0, every bridgeless cubic graph with no 3-edge-colouring has defect at least 3. In 2015, Steffen [1] proved that the Berge conjecture holds for cyclically 4-edge-connected cubic graphs with defect 3 or 4. We extend this result to all bridgeless cubic graphs irrespectively of their connectivity. Moreover, if the graph in question is cyclically 4-edge-connected, then four perfect matchings suffice, unless it is the Petersen graph. The result is best possible as there exists an infinite family of cubic graphs with cyclic connectivity 3 which have defect 3 but cannot be covered with four perfect matchings.

Keywords: Berge conjecture; coloring defect; cubic graph, perfect matching

MSC(2010): 05C70, 05C15

References

[1] E. Steffen. "1-*Factor and cycle covers of cubic graphs*", J. Graph Theory, **78**, 195–206, 2015.

A modular equality for Cameron-Liebler line classes in PG(n, q), $n \geq 7$ odd

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Abstract

In 1982 Cameron and Liebler tried to classify certain subgroup structures in PG$(3, q)$. As a byproduct they observed line classes that later adopted the name Cameron-Liebler line classes. These line classes were also generalized to line classes in PG(n, q). A Cameron-Liebler (CL) line class $\mathcal{L}$ is a set of lines in PG(n, q), $n \geq 3$, such that its characteristic vector $\chi_{\mathcal{L}}$ is a linear combination of the characteristic vectors of point-pencils. Typically, $\mathcal{L}$ has a parameter x that satisfies $|\mathcal{L}| = x\frac{q^n-1}{q-1}$. Note that it can be shown that x is required to be an integer if $n+1$ is even. In [2], it was proven that the parameter $x \in \mathbb{N}$ of a CL line class $\mathcal{L}$ in PG$(3, q)$ has to satisfy the equations $x(x-1) + 2m(m-x) \equiv 0 \pmod{2(q+1)}$. Here m denotes the number of lines of $\mathcal{L}$ in any plane or through any point. In recent work, see [1], a generalization of this modular equality was found for CL line classes $\mathcal{L}$ in PG(n, q), with $n \geq 7$ odd. Similarly, the parameter $x \in \mathbb{N}$ of the CL line class needs to satisfy $x(x-1) + 2m(m-x) \equiv 0 \pmod{q+1}$, with m equals the number of lines of $\mathcal{L}$ through any point.

MSC(2010): 05B20, 05B25

Keywords: Cameron-Liebler line classes; projective space; non-existence results

References

[1] J. De Beule and J. Mannaert. A modular equality for Cameron-Liebler line classes in projective and affine spaces of odd dimension. *Finite Fields Appl. Submitted.*

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New MRD codes from linear cutting blocking sets

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Joint work with: Daniele Bartoli and Alessandro Neri

Abstract

Minimal rank-metric codes or, equivalently, linear cutting blocking sets are characterized in terms of the second generalized rank weight, via their connection with evasiveness properties of the associated q-system. Using this result, we provide the first construction of a family of $\mathbb{F}_{q^m}$-linear MRD codes of length $2m$ that are not obtained as a direct sum of two smaller MRD codes. In addition, such a family has better parameters, since its codes possess strictly larger generalized rank weights than those of the previously known MRD codes. This shows that not all the MRD codes have the same generalized rank weights, in contrast to what happens in the Hamming metric setting.

Keywords: MRD codes; cutting blocking sets; q-systems, evasive subspaces; generalized rank weights

MSC(2010): 94B05, 51E20, 94B27

On Voloshin colorings in 3-hypergraph designs

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Joint work with: Paola Bonacini

Abstract

Let $P^{(3)}(2,4)$ be the hypergraph having $\{1,2,3,4\}$ as vertex set and $\{\{1,2,3\},$
$\{1,2,4\}\}$ as edge set. In this paper we consider vertex colorings of $P^{(3)}(2,4)$-designs in such a way any block is neither monochromatic nor polychromatic. We find bounds for the upper and lower chromatic numbers, showing also that these bounds are sharp. Indeed, for any admissible v there exists a $P^{(3)}(2,4)$-design of order v having the largest possible feasible set. Moreover, we study the existence of uncolorable $P^{(3)}(2,4)$-designs, proving that they exist for any admissible order $v \geq 28$, while for $v \leq 13$ any $P^{(3)}(2,4)$-design is colorable. Thus, a few cases remain open, precisely $v = 14, 16, 17, 18, 20, 21, 22, 24, 25, 26$.

Keywords: Coloring, Hypergraph, Design

MSC(2010): 05C15

References

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The cylinder conjecture and divisible codes

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Joint work with: Sascha Kurz

Abstract

The strong cylinder conjecture, as posed in a paper by Simeon Ball [1], is a simple-to-state conjecture, but has resisted all attacks so far. For any prime p, we can put it as follows: except for the set of points on p parallel lines in $\mathrm{AG}(3,p)$, are there other sets of p^2 points in $\mathrm{AG}(3,p)$ such that every plane intersects it in $0 \pmod p$ points? This conjecture builds on work of Rédei regarding factorizations of elementary abelian groups and has been generalized by Sascha Kurz and myself to the setting of divisible codes [2]. The latter setting provides a natural context to extend the conjecture to prime powers q and higher dimensions. We will discuss the historical context and the new coding-theoretical setting in which it appears. Surprisingly, it turns out that higher dimensional generalizations are equivalent to the original conjecture. We will see that the latter is true for small primes by combinatorial techniques, but fails for almost all prime powers, starting with $q = 8$. This shows that the combinatorial approach quickly reaches its limits and new techniques are necessary to advance us to a full solution.

Keywords: cylinder conjecture, divisible codes

MSC(2010): 05B25, 51D20, 51E22

References

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Highly edge-connected r-regular graphs without $r-2$ pairwise disjoint perfect matchings

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Joint work with: Eckhard Steffen

Abstract

In this talk we present infinite families of highly edge-connected r-regular graphs of even order which do not contain $r-2$ pairwise disjoint perfect matchings. When r is a multiple of 4, this result solves a problem stated by Thomassen in [1].

Keywords: Regular graphs; perfect matchings.

MSC(2010): 05C70

References

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On the existence of globally simple relative non-zero sum Heffter arrays

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Joint work with: Anita Pasotti

Abstract

In [1] Costa, Della Fiore and Pasotti introduced a class of partially filled arrays with entries in a cyclic group, called *non-zero sum Heffter arrays*. Whenever the partial sums of every column (from top to bottom) and of every row (from left to right) of an array are all distinct, the array is called *globally simple*. From a globally simple (relative) non-zero sum Heffter array one can obtain two cyclic orthogonal path decompositions of the complete (multipartite) graph.

In [2] we focus on completely filled square globally simple relative non-zero sum Heffter arrays, denoted as $NH_t(n)$. In particular, we give direct constructions for odd n and several values of t, completely solving the existence problem of a globally simple $NH_t(n)$ for every prime n and every admissible t.

Keywords: Heffter arrays; orthogonal cyclic path decompositions.

MSC(2010): 05B20, 05C38

References

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[2] L. Mella and A. Pasotti. "*Globally simple relative non-zero sum Heffter arrays*", in preparation.

On anti-Novák cycle systems

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Joint work with: Marco Buratti

Abstract

Novák's conjecture [3] states that any cyclic Steiner Triple System of order $v \equiv 1 \pmod 6$ can be obtained from a set of *disjoint* base blocks. Until recently, very little was known on the truth of this conjecture, but the recent paper by Feng, Horsley and Wang [2] contains significant progress: besides considering the original conjecture, the authors extend it to cyclic Steiner 2-designs, and more generally to cyclic 2-designs. Motivated by this work, in this talk we will consider instead a generalization to cyclic k-cycle systems: we show that in this setting the generalized conjecture is false for $k \geq 5$, construct some families of counterexamples which arise and discuss related problems.

Keywords: Cyclic Steiner triple systems, cyclic cycle systems, Novák's conjecture

MSC(2010): 05B07, 05B30

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[3] J. Novák, "*A note on disjoint cyclic Steiner triple systems*", in: Recent Advances in Graph Theory, Proc. Symp., Prague, 1974, 439–440 (1975).

Flag-transitive, point-imprimitive symmetric 2-designs

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Abstract

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ be a symmetric 2-(v, k, λ) design admitting a flag-transitive, point-imprimitive automorphism group G that leaves invariant a non-trivial partition Σ of $\mathcal{P}$. C. E. Praeger and S. Zhou [4] have shown that, there is a constant k_0 such that, for each $B \in \mathcal{B}$ and $\Delta \in \Sigma$, the size of $|B \cap \Delta|$ is either 0 or k_0. In this talk, which is based on the result contained in [2], we show that, if $k > \lambda(\lambda - 3)/2$ and $k_0 \geq 3$, $\mathcal{D}$ is isomorphic to one of the flag-transitive, point-imprimitive symmetric 2-designs with parameters $(45, 12, 3)$ or $(96, 20, 4)$ classified in [3] and in [1] respectively.

Keywords: Flag-transitive symmetric design; automorphism group.

MSC(2010): 05B05; 05B25; 20B25.

References

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Characterization of association schemes arising from finite geometry

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Abstract

One of the main goal within the theory of association schemes is, without any doubt, that of characterizing known association schemes by their parameters, even better if arising from finite geometry. This problem has its roots in a question originally posed by R.C. Bose [1] about strongly-regular graphs and partial geometries.

The Penttila-Williford scheme [3] and tha Van Dam-Martin-Muzychuk scheme [2] are characterized by their respective parameters.

Keywords: association schemes, generalized quadrangles

MSC(2010): 05E30, 51E12

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Carlitz-like identities

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Abstract

In 1971, Carlitz obtained the following symmetric identity [1]

$$(-1)^n \sum_{k=0}^{n} \binom{n}{k} B_{m+k} = (-1)^m \sum_{k=0}^{m} \binom{m}{k} B_{n+k}$$

involving the *Bernoulli numbers*. This identity has been proved and generalized in several ways by several authors. For instance, it has been generalized to the *Bernoulli polynomials*

$$(-1)^n \sum_{k=0}^{n} \binom{n}{k} B_{m+k}(x)\, y^{n-k} = (-1)^m \sum_{k=0}^{m} \binom{n}{k} B_{n+k}(1-x-y)\, y^{m-k}$$

and, more in general, to the *q-Bernoulli numbers* and *polynomials*.

Starting from these results, we obtain [2] a *Carlitz-like identity* for a large class of polynomials. In particular, such an identity is satisfied by many classical polynomials, such as the *generalized Bernoulli* and *Euler polynomials*, the *generalized Hermite polynomials*, and the *Nörlund polynomials*.

Keywords: combinatorial sums, umbral calculus, Appell polynomials, Sheffer matrices.

MSC(2010): 05A19, 05A40

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Simplicity conditions for binary orthogonal arrays

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Joint work with: Claude Carlet and Rebeka Kiss

Abstract

Let N, t, k be positive integers, $t \leq k$, and S a finite set of cardinality s. An $N \times k$ array A with entries from S is said to be an *orthogonal array with s symbols, strength t, and index λ,* if every $N \times t$ subarray of A contains each t-tuple based on S exactly λ times as a row. An orthogonal array is *simple,* if the rows are distinct. Supports of t-th order CI-functions give simple binary orthogonal arrays with strength t, if their elements are written as rows.

In the theory of orthogonal arrays, the main question is to give – for given number of columns, symbols and strength – the minimum value of N for which a simple orthogonal array exists with N rows. We will denote this value by $F^*(k, s, t)$. If we do not require simplicity, the minimum value of rows is denoted by $F(k, s, t)$. This problem is very hard even for the smallest parameters $s = t = 2$.

In [1], Carlet and Guilley asked the the following question: *Is $F^*(k, 2, t)$ a monoton non-decreasing function when k grows and t remains fixed?* In our talk, we use Rao's Bound to give a sufficient condition for an orthogonal array to be simple. We apply this result to compute the minimum number of rows in simple binary orthogonal arrays of strength 4 and 5.

MSC(2010): 05B15, 94B65

Keywords: Correlation-immune Boolean functions; orthogonal arrays; Rao's bound

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Looking for additive Steiner 2-designs

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Joint work with: Marco Buratti

Abstract

A 2-(v, k, λ) design is additive under an abelian group G if, up to isomorphism, the point-set is a subset of G and each block sums to zero. This definition was the starting point of an interesting theory developed in [3]. What we found intriguing was that additive Steiner 2-designs appeared in short supply; the only known classes were the point-line designs of AG(n, q) if $q > 2$, and of PG(n, q) if either n or q is 2. Also, the only known sporadic example was the famous 2-analog of a 2-$(13, 3, 1)$ design found in [1] which is an additive 2-$(8191, 7, 1)$ design. Thus all known Steiner 2-designs had block-size k equal to a prime power or a prime power plus one. Even though we proved their existence for any $k \not\equiv 2 \pmod 4$ not of the form $2^n 3$, it is disappointing that the values of v are huge [2]. On the other hand we just found a pair of "small" additive Steiner 2-designs not lying in the above mentioned classes; a 2-$(40, 4, 1)$ design which is additive under $\mathbb{Z}_3^4$ and a 2-$(124, 4, 1)$ design which is additive under $\mathbb{Z}_5^3$.

Keywords: Additive design; Steiner 2-design; difference methods.

MSC(2010): 05B05, 05B10

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On some linear sets in finite projective spaces and the associated codes

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Joint work with: Olga Polverino, Paolo Santonastaso and Ferdinando Zullo

Abstract

The relationship between subsets of points of finite projective spaces and linear codes is a motivation for the study of such objects that are interesting in themselves and therefore have attracted the attention of finite geometers, combinatorists and coding theorists. In this talk I will present some results concerning linear codes with few weights arising from some special linear sets of finite projective spaces, in particular linear sets of the desarguesian projective plane $\mathrm{PG}(2, q^n)$. The talk is based on the papers [1, 2].

Keywords: Linear set; codes with few weights; MRD–code.

MSC(2010): 05B25, 51E20

References

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Comparison between the eccentric connectivity index and first Zagreb index of graph

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Joint work with: Prof. Kinkar Chandra Das

Abstract

For a connected graph G, the eccentric connectivity index and the first Zagreb index of G are defined as $\xi^c(G) = \sum_{v_i \in V(G)} d_i \varepsilon_i$ and $M_1(G) = \sum_{v_i \in V(G)} d_i^2$, respectively, where d_i is the degree of v_i in G and ε_i denotes the eccentricity of vertex v_i in G. Recently, Das and Trinajstić (2011)[1], compared the eccentric connectivity index and Zagreb indices for chemical tree and molecular graphs. However, the comparison between the eccentric connectivity index and Zagreb indices, in the case of general trees and general graphs, is very hards and remains unsolved till now. In this paper we compare the eccentric connectivity index and the first Zagreb index of graphs, where $\ominus(T) = \xi^c(T) - M_1(T)$ for any tree T. As a results, we proved that $\ominus(T)$ is minimum for T is caterpillar.

Keywords: Tree graph; Caterpillar graph; Eccentric connectivity index (ξ^c); First Zagreb index (M_1).

MSC(2010): 00X01, 01X02

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Strong blocking sets and minimal codes from graphs

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Abstract

Strong blocking sets are special sets of n points in $\mathrm{PG}(k-1,q)$, such that their intersection with each hyperplane generates the hyperplane itself. They have recently been shown to correspond to k-dimensional minimal linear codes in $\mathbb{F}_q^n$, even though they were originally introduced as a tool to construct families of covering codes. In this talk we propose a general method to construct small strong blocking sets starting from a set of points in $\mathrm{PG}(k-1,q)$ and a graph with special connectivity properties.

Keywords: blocking sets; strong blocking sets; rook graph; minimal linear codes.

MSC(2010): 51E21, 94B05, 05C69, 05C40

Complementary prisms: their cores, automorphism groups, isoperimetric numbers, hamiltonian properties, etc.

Marko Orel

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Abstract

Given a finite simple graph Γ its complementary prism is the graph $\Gamma\bar{\Gamma}$ that is obtained from Γ and its complement $\bar{\Gamma}$ by adding a perfect matching, where each its edge connects two copies of the same vertex in Γ and $\bar{\Gamma}$. If Γ is the pentagon, then $\Gamma\bar{\Gamma}$ is the Petersen graph, which is known to be a *core*, i.e. all its endomorphisms are automorphisms. The talk will address the following question: Which properties on Γ guaranty that its complementary prism $\Gamma\bar{\Gamma}$ is a core? The main focus will be on vertex-transitive self-complementary graphs Γ, since in this case $\Gamma\bar{\Gamma}$ is vertex-transitive (but not a Cayley) graph, and on strongly regular self-complementary graphs. In addition, the automorphism group and the isoperimetric number of $\Gamma\bar{\Gamma}$ will be described for arbitrary simple graph Γ. Hamiltonian properties of regular complementary prisms will be addressed as well. Some of the proofs (and open problems) involve graph spectrum and the Lovász theta function. The talk will be based on [1].

Keywords: graph homomorphism; core; automorphism group; self-complementary graph; vertex-transitive graph; strongly regular graph; non-Cayley graph; isoperimetric number; graph spectrum

MSC(2010): 05C60, 05C76, 05E30, 05C45

References

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Ghosts arising from polynomials

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Joint work with: Marco Della Vedova (UniBG) and Silvia Pianta (UniCatt)

Abstract

Given a point P in $\mathrm{PG}(n, q)$, its Rédei factor is the linear polynomial in $n + 1$ variables, whose coefficients are the point coordinates. The power sum polynomial G^S associated to a multi-subset S of the projective plane $\mathrm{PG}(2, q)$ is the sum of the $(q - 1)$-th powers of the Rédei factors of the points of S [3]. The classification of multi-subsets having the same power sum polynomial bases on the determination of those multi-subsets associated to the zero polynomial, called ghosts. In fact, two multi-subsets S_1 and S_2 such that $G^{S_1} = G^{S_2}$ "differ" by a ghost Z, namely, $S_2 = S_1 \uplus_p Z$, where $\uplus_p$ is the multiset sum modulo p (the field characteristic).

In this talk we investigate the space of ghosts, compute its dimension and characterize some classes of ghosts. Moreover, we explicitly enumerate ghosts for planes of small order. The present talk is based on [1, 2].

Keywords: Ghost; multiset sum; power sum polynomial; projective plane

MSC(2010): 51E15, 51E22, 11T06, 05B25

References

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New result on permutation binomials

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Joint work with: Xiang-dong Hou

Abstract

After a brief review of existing results on permutation binomials of finite fields, we introduce the notion of equivalence among permutation binomials (PBs) and describe how to bring a PB to its canonical form under equivalence. We then focus on PBs of $\mathbb{F}_{q^2}$ of the form $X^n(X^{d(q-1)}+a)$, where n and d are positive integers and $a \in \mathbb{F}_{q^2}^*$. Our contributions include two nonexistence results: (1) If q is even and sufficiently large and $a^{q+1} \neq 1$, then $X^n(X^{3(q-1)}+a)$ is not a PB of $\mathbb{F}_{q^2}$. (2) If $2 \leq d \mid q+1$, q is sufficiently large and $a^{q+1} \neq 1$, then $X^n(X^{d(q-1)}+a)$ is not a PB of $\mathbb{F}_{q^2}$ under certain additional conditions. (1) partially confirms a recent conjecture by Tu et al. (2) is an extension of a previous result with $n = 1$.

Keywords: finite field, Hasse–Weil bound, permutation binomial

MSC(2010): 11T06, 11T55, 14H05

References

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Comparing balanced $\mathbb{Z}_v$-sequences obtained from ElGamal function to random balanced sequences

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Joint work with: Lucas Perin and Brett Stevens

Abstract

We investigate the randomness properties of sequences in $\mathbb{Z}_v$ derived from permutations in $\mathbb{F}_p^*$ using the remainder function. Motivated by earlier studies with a cryptographic focus, we compare sequences constructed from the ElGamal function $x \to g^x$ for $x \in \mathbb{Z}_{>0}$ and g a primitive element of $\mathbb{F}_p^*$, to sequences constructed from random permutations of $\mathbb{F}_p^*$.

We prove that sequences obtained from ElGamal have maximal period and behave similarly to random permutations with respect to the balance and run properties of Golomb's postulates for pseudo-random sequences. Additionally we show that they behave similarly to random permutations for the tuple balance property. This requires some work to determine properties of random balanced periodic sequences. In general, for these properties and excepting for very unlikely events, the ElGamal sequences behave the same as random balanced sequences.

Keywords: Randomness in finite fields; ElGamal function; Golomb postulates.

MSC(2010): 12E20, 11T06, 11T71

The graph of non-degenerate linear codes

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Abstract

Let $\Gamma(n,k)_q$ be the graph consisting of all non-degenerate linear $[n,k]_q$ codes with $1 < k < n-1$; two such codes are connected by an edge in this graph if they have the maximal number of the same codewords. This is a subgraph of the Grassmann graph formed by k-dimensional subspaces of an n-dimensional vector space over the q-element field. All automorphisms of the Grassmann graph are known; they are induced by semilinear automorphism of the vector space or seimilear isomorphisms to the dual vector space and the second possibility is realized only for $n = 2k$. We prove the following

- If $q \geq 3$ or $k \neq 2$, then every isomorphism of $\Gamma(n,k)_q$ to a subgraph of the Grassmann graph can be uniquely extended to an automorphism of the Grassmann graph. In other words, a subgraph of the Grassmann graph isomorphic to $\Gamma(n,k)_q$ is unique up to automorphism.
- In the case when $q = k = 2$, the Grassmann graph contains subgraphs isomorphic to $\Gamma(n,k)_q$ and such that isomorphisms between these subgraphs and $\Gamma(n,k)_q$ cannot be extended to automorphisms of the Grassmann graph.

Keywords: Grassmann graph; linear code

MSC(2010): 94B05

On non-zero sum Heffter arrays

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Joint work with: Simone Costa, Stefano Della Fiore

Abstract

In [2] we introduced a new class of partially filled arrays that, as Heffter arrays [1], are related to difference families, graph decompositions and biembeddings. A *non-zero sum Heffter array* $\mathrm{NH}(m, n; h, k)$ is an $m \times n$ partially filled array with entries in $\mathbb{Z}_{2nk+1}$ such that: each row contains h filled cells and each column contains k filled cells; for every $x \in \mathbb{Z}_{2nk+1} \setminus \{0\}$, either x or $-x$ appears in the array; the sum of the elements in every row and column is different from 0 (in $\mathbb{Z}_{2nk+1}$). Existence results and connections with other topics will be presented.

Keywords: Heffter array, orthogonal cyclic path decomposition

MSC(2010): 05B20, 05C38

References

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Clifford-like parallelisms and their automorphisms

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Joint work with: H. Havlicek and S. Pianta

Abstract

It is well known that the three dimensional real projective space $\mathrm{PG}(3,\mathbb{R})$ can be endowed with two projectively equivalent parallelisms, namely the *left* and *right Clifford parallelisms*. In [1] we extended this definition of Clifford parallelisms to the case of a commutative field F of any characteristic and we proposed a technique for the construction of a class of "*Clifford-like*" regular parallelisms of $\mathrm{PG}(3,F)$ which has no counterpart in the classical case.

Here we present necessary and sufficient conditions for the existence of Clifford-like parallelisms that are not Clifford and a description of their automorphism group by comparing it with the automorphism group Γ of the left Clifford parallelism. Since the linear part of Γ is the same for all Clifford-like parallelisms which can be associated to it, we study the action of Γ on parallel classes in order to characterise Clifford parallelisms among Clifford-like ones (See [2, 3]).

Keywords: Clifford parallelism; Clifford-like parallelism; Projective double space; quaternion skew field; Automorphism

MSC(2010): 51A15, 51J15

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On m-ovoids of elliptic quadrics

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Joint work with: A.L. Gavrilyuk and K. Metsch

Abstract

An m-ovoid of a finite polar space $\mathcal{P}$ is a set $\mathcal{O}$ of points such that every maximal subspace of $\mathcal{P}$ contains exactly m points of $\mathcal{O}$. In this talk it will be shown that in the case when $\mathcal{P}$ is an elliptic quadric $\mathcal{Q}^-(2r+1,q)$ of rank r in $\mathbb{F}_q^{2r+2}$, an m-ovoid exists only if m satisfies a certain modular equality, which depends on q and r. This condition rules out many of the possible values of m. A characterization of the m-ovoids of $\mathcal{Q}^-(7,q)$ for $q=2$ and $(m,q)=(4,3)$ will also be discussed.

Keywords: Elliptic quadric; m-ovoid.

MSC(2010): 51E20, 05B25, 51A50

References

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Small configurations and some structure theorems for Steiner triple systems

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Abstract

An interesting and elegant paper on Steiner triple systems was published in 2010 [1], where the authors characterized the classes of Hall triple systems, affine Hall triple systems and affine Steiner triple systems in terms of avoidance of certain small configurations. In 2012 some other authors [2] improved and simplified two of the characterizations in [1]. In this talk we present some of the results of a paper in preparation, one of which is an improvement of the characterization of affine Steiner triple systems in [1]. Moreover, we simplify the proof of the other characterizations in [1] and [2], and present new characterizations of Hall triple systems, affine Hall triple systems and projective triple systems in terms of various small configurations, such as Pasch configurations, mitres, grids, and the configurations C_S^1, C_S^2, and C_{14}.

Keywords: Steiner triple system, Affine triple system, Hall triple system, forbidden configuration, Pasch configuration, mitre.

MSC(2010): 05B07, 05B05, 05B30.

References

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Linear sets with points of weight larger than 1

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Joint work with: B.Csajbók and G.Marino

Abstract

Let V be an r-dimensional vector space over $\mathbb{F}_{q^n}$. A point set L of $\mathrm{PG}(V, q^n) = \mathrm{PG}(r-1, q^n)$ is called an $\mathbb{F}_q$-*linear set* of rank k if it is defined by the non-zero vectors of a k-dimensional $\mathbb{F}_q$-vector subspace U of V, i.e.

$$L = L_U = \{\langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} : \mathbf{u} \in U \setminus \{\mathbf{0}\}\}.$$

For a point $P = \langle \mathbf{z} \rangle_{\mathbb{F}_{q^n}} \in \mathrm{PG}(V, q^n)$ the *weight* of P with respect to the linear set L_U is $w_{L_U}(P) := \dim_q(\langle \mathbf{z} \rangle_{\mathbb{F}_{q^n}} \cap U)$.

Let L_U be a linear set such that $w_{L_U}(P) > 1$ for every $P \in L_U$. We determine when L_U is a linear set over a field $\mathbb{F}_{q^d}$, $d|n, d > 1$.

Keywords: Linear sets; Finite fields

MSC(2010): 51E23, 12E20

Automorphisms of generalized Grassmann graphs

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Joint work with: Mark Pankov and Mariusz Żynel

Abstract

The Grassmann graph is the adjacency graph of the Grassmann space. Classical Chow's theorem describes automorphisms of this graph.

The Grassmannian of m-dimensional subspaces of a complex Hilbert space can be naturally identified with the conjugacy class of rank-m projections. In [1] we extend the adjacency relation from a conjugacy class of projections to a conjugacy class $\mathcal{C}$ of finite-rank self-adjoint operators, and study the corresponding generalized Grassmann graph $\Gamma_{\mathcal{C}}$. Using an approach based on Johnson graphs, in [2] we show that, under the assumption that operators from $\mathcal{C}$ have more than two eigenvalues, every automorphism of $\Gamma_{\mathcal{C}}$ is induced by a unitary or anti-unitary operator up to a permutation of eigenspaces with the same dimensions.

Keywords: Grassmann graph; conjugacy class of finite rank self-adjoint operators; graph automorphism; preserver problems

MSC(2010): 05E18, 47B15, 81P10

References

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Minimum size linear sets and critical pairs

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Joint work with: Vito Napolitano, Paolo Santonastaso and Ferdinando Zullo

Abstract

Linear sets have been deeply studied and there is still a massive attention on them, as they have been used to construct and classify several objects. In this talk we will deal with linear sets of **minimum size** in $\mathrm{PG}(1, q^n)$ [3]. Examples of these linear sets have been found by Lunardon and Polverino (2000) and, more recently, by Jena and Van de Voorde in [4]. However, classification results for minimum size linear sets of rank k are known only for $k \leq 5$. In this talk we will provide classification results for linear sets of minimum size when n is prime, answering to a question posed in [4]. Then we construct new examples when n is not prime. The main tool relies on studying pairs of subspaces, *critical pairs*, attaining the equality in the linear analogue of Cauchy-Davenport's theorem [1, 2]. The talk is based on the paper arXiv:2201.02003.

Keywords: Linear set; minimum size; critical pair.

MSC(2010): 05B25, 51E20, 11P70, 12F99

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4-sets in Union-Closed Families

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Joint work with: Kenan Wood

Abstract

Frankl's conjecture (FC) states that for every family of sets that is closed under union, there is an element that belongs to at least half the sets in the family. Local configurations are families of sets whose presence ensures that FC holds for any family which is closed under union and contains them. It is easy to see that a single 1-set or a single 2-set are local configurations. Unfortunately a single 3-set is not a local configuration. A natural question is to consider minimal number of 3-sets that are local configurations. Recently we gave a polyhedral version of the main theorem for the classification of local configurations [3], and the resulting computational framework settled the question of 3-sets in union-closed families [2]. In this talk we will discuss preliminary results on minimal collections of 4-sets that are local configurations, improving on results in Morris [1]. We will also discuss implications for improving lower bounds on the size of the ground set for FC.

Keywords: Frankl's conjecture; Union-Closed Families; Local Configurations.

MSC(2010): 05D05, 05-08

References

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49,487,367,289: On enumeration and computational construction of groups of order 1024

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Joint work with: Josue Avila Artavia

Abstract

In 2000, H.U. Besche, B. Eick, and E.A. O'Brien reported that there were 49,487,365,422 groups of order 1024 – which accounts for more than 99% of groups of order at most 2000. This number, proposed via an amalgamation of computational group theory and computer engineering, remained unchallenged for 2 decades until a recent correction by D. Burrell suggesting that there are actually 49,487,367,289 such groups. In this work, we review the primary counting method to arrive at these figures – the p-group generation algorithm, and provide a unified set of proofs for its underlying theory, which was previously scattered about in folklore. We then demonstrate how to efficiently construct groups of order 1024 using GAP and parallel programming. These techniques provide a framework for developing a complete library of 1024 groups, which is the only "small" order collection absent from the well-known Small Groups super-library in GAP.

Keywords: computational group theory; small groups; parallel programming; GAP

MSC(2010): 20C40, 20B40

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Decomposition of cubic graphs with cyclic connectivity 6 and beyond

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Joint work with: Edita Máčajová

Abstract

Cyclic edge-connectivity, that is the minimum number of edges we need to remove to separate two cycles, is an important invariant of graphs, especially with regards to some famous conjectures. A component of a cyclically k-edge-connected cubic graph separated by a cycle separating k-edge-cut is called a *cyclic k-part*. We discuss how a cyclic k-part can be completed to a cyclically k-edge-connected cubic graph. Our work extends the results of Andersen et al. [1] for the case $k = 4$, and our recent results [2] for the case $k = 5$.

We prove that if we join two cyclic k-parts, both different from the k-cycle, by interconnecting their degree 2 vertices without creating a cycle of length shorter than k, then the resulting cubic graph is cyclically k-edge-connected. Using this result we show that each cubic graph with cyclic connectivity 6 can be decomposed into two cyclically 6-edge-connected graphs, where its cyclic 6-parts are completed by adding 8 additional vertices forming two 6-cycles that share a path of length three.

Keywords: cubic graphs, cyclic connectivity, decomposition, girth

MSC(2010): 05C40

References

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Ternary Linear Codes and Blocking Sets in PG$(5, 3)$

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Joint work with: Ivan Landjev and Emeliyan Rogachev

Abstract

We consider the problem of determining the exact value of $n_q(k, d)$ defined as the minimal length of a linear code of fixed dimension k, and fixed minimum distance d over the field $\mathbb{F}_q$. For fixed k and q this problem is a finite one since Griesmer codes exist for all sufficiently large d. A closely related geometric problem, formulated by Hamada some thirty years ago, is to characterize all blocking (multi)sets in PG(t, q) with parameters $\left(\sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_{\alpha+1}, \sum_{\alpha=0}^{t-1} \varepsilon_\alpha v_\alpha\right)$, where $v_k = (v^k - 1)/(v - 1)$. In this paper, we characterize all blocking sets with parameters $(v_t + 2v_{t-1} + \varepsilon, v_{t-1} + 2v_{t-2})$ in PG(t, q), where $t \geq 3$, $q \geq 3, \varepsilon \in \{0, 1, 2\}$.

For $q = 3$, the exact value of $n_3(k, d)$ is known for all d, and all $k \leq 5$. In a recent paper, [1] it was reported that the exact value of $n_3(6, d)$ is known for all but 70 values of d. The characterization of the blocking sets with the above parameters is used to prove the nonexistence of the hypothetical Griesmer codes for $q = 3$, $k = 6$, and $d = 100, 343, \ldots, 346$. This solves five open cases in the tables from [1].

Keywords: optimal linear codes; Griesmer bound; blocking sets; the main problem of coding theory

MSC(2010): 94B05, 94B27, 05B25, 51E20

References

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Edge-transitive Nest graphs of twice odd order

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Joint work with: István Kovács

Abstract

A graph admitting an automorphism with two orbits of the same length is called a bicirculant. The Nest graphs are the members of a 5-parameter family of hexavalent bicirculants, which was introduced by Jajcay et al. (Electron. J. Combin., 2019). We show that, up to isomorphism, the complement of the Petersen graph is the only edge-transitive Nest graph whose order is twice an odd number.

The main result is the following:

Theorem 1. *If $\mathcal{G} = \mathcal{N}(n; a, b, c; k)$ is an edge-transitive Nest graph for an odd number n, then $\mathcal{G}$ is isomorphic to the complement of the Petersen graph.*

Keywords: edge-transitive, Nest graph, bicirculants

References

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On subspace designs

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Joint work with: Ferdinando Zullo

Abstract

In this talk, we will investigate the theory of subspace designs, which have been originally introduced by Guruswami and Xing in [1] to give the first construction of positive rate rank-metric codes list decodable beyond half the distance.

A collection of $\mathbb{F}_q$-subspaces $U_1, \dots, U_t \subseteq V(k, q^m)$ is called an $(s, A)_q$-**subspace design** if $\sum_{i=1}^{t} \dim_{\mathbb{F}_q}(U_i \cap W) \leq A$, for every $\mathbb{F}_{q^m}$-subspace $W \subseteq V(k, q^m)$ of dimension s.

When $t = 1$, $(s, A)_q$-subspace designs coincide with the notion of evasive subspace, originally introduced by Pudlák and Rödl. We will provide bounds involving the dimension of the subspaces forming a subspace design and the parameters of the ambient space, showing optimal constructions with respect to these bounds. Then we will also introduce two dualities relations among them. Special attention will be paid to $(s, s)_q$-subspace designs that generalize the notion of s-scattered subspace introduced by Csajbók, Marino, Polverino and Zullo. We will show that, for certain values of s, they are associated with linear maximum sum-rank metric codes. Morover, in the case $s = 1$, we will provide several examples and we will get families of two-intersection sets with respect to hyperplanes (and hence two-weight linear codes).

MSC(2010): 51E23, 94B05

Keywords: Subspace design; subspace evasive subspace; sum-rank metric code

References

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Designs in finite general linear groups

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Joint work with: Alena Ernst

Abstract

It is known that the notion of a transitive subgroup of a permutation group G extends naturally to subsets of G. This talk is about subsets of the general linear group $\mathrm{GL}(n,q)$ acting transitively on flag-like structures, which are common generalisations of t-dimensional subspaces of $\mathbb{F}_q^n$ and bases of t-dimensional subspaces of $\mathbb{F}_q^n$. I shall discuss structural characterisations of transitive subsets of $\mathrm{GL}(n,q)$ using the character theory of $\mathrm{GL}(n,q)$ and interprete such subsets as designs in the conjugacy class association scheme of $\mathrm{GL}(n,q)$. While transitive subgroups of $\mathrm{GL}(n,q)$ are quite rare, it will be shown that, for all fixed t, there exist nontrivial subsets of $\mathrm{GL}(n,q)$ that are transitive on linearly independent t-tuples of $\mathbb{F}_q^n$, which also shows the existence of nontrivial subsets of $\mathrm{GL}(n,q)$ that are transitive on more general flag-like structures. These results can be interpreted as q-analogs of corresponding results for the symmetric group.

Keywords: Association schemes, designs, finite general linear groups, transitivity

MSC(2010): 05B99, 05E30, 20C33

Cyclic Line Spreads and Linear Spaces

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Joint work with: Cian Jameson

Abstract

There has been much progress towards classifying *linear spaces* that possess a flag-transitive automorphism group. However, a complete classification is not available, as the case in which the automorphism group is a subgroup of one-dimensional affine transformations remains open; in particular, linear spaces constructed from *spreads* possessing a transitive automorphism group.

In [1], Pauley and Bamberg constructed new flag-transitive linear spaces via spreads upon which a cyclic group acts transitively. and provided a condition for such spreads to exist in terms of an associated polynomial.

In this talk, we will present our work on describing and classifying the polynomials that give rise to the desired spreads and linear spaces. We will focus on the case of cubic polynomials, corresponding to cyclic line spreads in $\mathrm{PG}(5, q)$.

Keywords: spread, linear space, transitive, automorphism

MSC(2010): 05B25, 51E23

References

[1] M. Pauley, J. Bamberg. "*A construction of one-dimensional affine flag-transitive linear spaces*", Finite Fields and their Applications, (14):537–548, 2008.

Snarks with resistance n and flow resistance $2n$

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Joint work with: Imran Allie, Edita Máčajová

Abstract

We examine the relationship between two measures of uncolourability of cubic graphs – their resistance and flow resistance. The resistance of a cubic graph G, denoted by $r(G)$, is the minimum number of edges whose removal results in a 3-edge-colourable graph. The flow resistance of G, denoted by $r_f(G)$, is the minimum number of zeroes in a 4-flow on G. Fiol et al. [1] made a conjecture that $r_f(G) \leq r(G)$ for every cubic graph G. We disprove this conjecture by presenting a family of cubic graphs G_n of order $34n$, where $n \geq 3$, with resistance n and flow resistance $2n$. For $n \geq 5$ these graphs are nontrivial snarks.

Keywords: edge colouring; nowhere-zero flow; snark

MSC(2010): 05C70, 05C15

References

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Partial ovoids of symplectic and Hermitian polar spaces, large cocliques in the collinearity graph and applications

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Joint work with: Michela Ceria, Jan De Beule and Francesco Pavese

MSC(2010): 05B25, 05D10, 51A50

Abstract

Let $\mathcal{P}$ be a finite classical polar space of $PG(n,q)$. An *ovoid* $\mathcal{O}$ of $\mathcal{P}$ is a set of points of $\mathcal{P}$ such that every generator of $\mathcal{P}$ has exactly one point in common with $\mathcal{O}$. In the case when $\mathcal{P}$ does not admit ovoids, it is possible to introduce the notion of *partial ovoid*, such that every generator meets the ovoid in at most one point. In this case, the question about the size of the largest (maximal) partial ovoid naturally arises.

The collinearity graph $\Gamma_P = (V, E)$ of a polar space is the graph whose vertex set V is the set of points of $\mathcal{P}$ and in which adjacency belongs from collinearity. Large partial ovoids in generalized quadrangles correspond to large cocliques in the corresponding collinearity graphs.

In this talk some constructive lower bounds on the sizes of the largest partial ovoids of the symplectic polar space $W(3,q)$, q odd square and $q \neq 3^h$ and $W(5,q)$ and of the Hermitian polar space $H(4,q^2)$, $q \neq 3^h$, $H(6,q^2)$ and $H(8,q^2)$ will be discussed. Large cocliques in $\Gamma_{W(3,q)}$ have applications in Ramsey theory, too.

Keywords: Partial ovoids; symplectic polar spaces; Hermitian polar spaces

References

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Rigidity and flexibility of rod configurations

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Joint work with: Signe Lundqvist and Lars-Daniel Öhman

Abstract

A rod configuration is a geometric configuration of points and lines in Euclidean space together with a notion of motion for which the lines are rigid bodies. More precisely, a motion of a rod configuration is a motion of the points such that the distances between collinear points are preserved. Any rod configuration is moved by the Euclidean motions. Rod configurations that only admit the Euclidean motions are called rigid.

In this talk I will give new examples of flexible rod configurations and I will explain how to combinatorially characterize the incidence geometries that give rise to rod configurations in the plane that are infinitesimally rigid in sufficiently generic position.

Keywords: Combinatorial rigidity, rod configuration

MSC(2010): 05B30, 14N20, 52C25

References

[1] Signe Lundqvist, Klara Stokes, Lars-Daniel Öhman "*Exploring the infinitesimal rigidity of planar configurations of points and rods*", arXiv: 2110.07972.

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Hamiltonian graphs in Abelian 2-groups

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Abstract

We analyze a graph G with vertices that are subgroups of $\mathbb{Z}_2^k$ isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$. Two vertices are adjacent if they, as subgroups, have nontrivial intersection. We show that such a graph is $6(2^{k-2}-1)$-regular. In such cae, a classical theorem by Ore proves that a graph is Hamiltonian if the degree of any vertex is at least one half of the number of vertices. Ore's theorem is applicable for $k \in \{3,4\}$. Nevertheless, we manage to construct a Hamiltonian cycle for $k=5$. Our construction uses orbits of one $\mathbb{Z}_2^4$ group under an action of an automorphism of order 31. It is highly likely that this approach could be generalized for $k>5$.

Keywords: Hamiltonian Graph; Graph; Elementary Abelian Group; Subgroup; Group Ring

MSC(2010): 05C45, 13A50

References

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H-colorings in cubic and r-regular graphs

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Joint work with: G. Mazzuoccolo and J.P. Zerafa

Abstract

Let H and G be graphs: an H-coloring of G is a map $f : E(G) \to E(H)$ such that for any vertex $v \in V(G)$ there exists a unique vertex $u \in V(H)$ with $f(\partial_G(v)) = \partial_H(u)$, where $\partial_G(v)$ denotes the set of edges incident to the vertex v in the graph G. If G admits an H-coloring we say that H colors G. The question if there exists a graph H that colors every bridgeless cubic graph G is addressed directly by the Petersen-coloring conjecture, which poses H equal to the Petersen graph. Actually it has been shown that if the Petersen-coloring conjecture is true, the Petersen graph is the unique connected bridgeless cubic graph H which can color all the bridgeless cubic graphs. In this talk we survey some known results on H-colorings of graphs, considering several different assumptions on H and G, and provide some new results concerning uniqueness of H in the above sense.

Keywords: Cubic Graph; Petersen Colouring Conjecture; Regular Graph; Multigraph.

MSC(2010): 05C15, 05C70

References

[1] A. Hakobyan, V. Mkrtchyan. "S_{12} *and* P_{12}*-colorings of cubic graphs*", Ars Math. Contemp., **17**:431–445, 2019.

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A generalization of Bring's curve in any characteristic

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Joint work with: Gábor Korchmáros and Stefano Lia

Abstract

Bring's curve is well known from classical geometry as being the curve with the largest automorphism group among all genus 4 complex curves. In this talk, I will describe a natural generalization of Bring's curve valid over any field of zero or positive characteristic, that is the algebraic variety defined as the intersection of the projective algebraic hypersurfaces of homogeneous equations $x_1^k + \cdots + x_m^k = 0$ with $1 \leq k \leq m-2$. I will also point out a connection with the previous work of Rédei [1], as well as with a more recent result of Rodríguez Villegas, Voloch and Zagier [2] on plane curves attaining the Stöhr-Voloch bound, and the regular sequence problem for systems of diagonal equations introduced by Conca, Krattenthaler and Watanabe [3].

Keywords: Algebraic curves, positive characteristic, automorphism groups.

MSC(2010): 14H37, 14H05

References

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Infinite graph factorizations

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Joint work with: Simone Costa

Abstract

Over the past few years, there has been considerable interest in classic designs on an infinite set of points, mainly with block size three [1, 5]. However, only lately the general problems of factorizing infinite graphs, or finding a resolution of infinite-graph decompositions have been considered [2, 3].

In this talk, we outline some recent results [3, 4] and emphasize some open questions on the existence of factorizations of the Rado graph and infinite Cayley graphs.

Keywords: Graph factorization; Rado graph; Infinite (Cayley) graph; Regular 1-factorization

MSC(2010): 05C63, 05C70

References

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Weakly p-self-orthogonal designs and LCD codes

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Joint work with: Vedrana Mikulić Crnković

MSC(2010): 05E18, 94B05, 20D08, 05B05, 05E30

Abstract

A 1-design is weakly p-self-orthogonal if all the block intersection numbers gives the same residue modulo p. In [1], we analyze extensions of the incidence matrix, orbit matrix and submatrices of orbit matrix of a weakly p-self-orthogonal 1-design in order to construct self-orthogonal codes.

A linear codes is called LCD code if the intersection with its dual code is trivial. Matrix G generates an LCD code if and only if $\det(G \cdot G^T) \neq 0$ (see [3]). We extend the methods of construction described in [1] in order to construct LCD codes over finite fields. We use suitable extensions of incidence matrix, orbit matrices and submatrices of orbit matrices in order to construct LCD codes over finite field. We will present examples of LCD codes constructed from weakly p-self-orthogonal designs obtained from groups using construction described in [2].

Keywords: design, self-orthogonal design, LCD code

References

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A large family of maximum scattered linear sets of $PG(1, q^n)$ and their associated MRD codes

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Joint work with: G. Longobardi, G. Marino and Y. Zhou

Abstract

The concept of linear set in projective spaces over finite fields was introduced by Lunardon [1] and it plays central roles in the study of blocking sets, semifields, rank-metric codes, and other geometric structures. A linear set with the largest possible cardinality and rank, is called maximum scattered.

Despite two decades of study, there are only a limited number of maximum scattered linear sets of a line $PG(1, q^n)$. In this talk, we will exhibit a family of new maximum scattered linear sets of $PG(1, q^n)$ for any even $n \geq 6$ and odd q, which is considerably larger than classes known so far. Moreover, we will elaborate on maximum rank-distance codes ensuing from the constructed linear sets.

Keywords: Linear Set, Rank Distance Code, Linearized Polynomial

MSC(2010): 11T06, 11T71

References

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The Effect of Symmetry-preserving Operations on the 3-Connectedness of Embedded Graphs

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Abstract

Symmetry-preserving operations have been studied for a very long time. However only recently a general approach was presented to describe all 'local (orientation-preserving) symmetry-preserving operations, l(op)sp-operations for short [1]. Many well-known and used operations such as the dual, ambo and truncation are lopsp-operations. For plane graphs it is known that all lopsp-operations preserve 3-connectedness, but for graphs of higher genus that is not the case. The most striking example of a lopsp-operation that can greatly reduce the connectivity of a graph is the dual. We use the new description of symmetry-preserving operations to determine for any lopsp-operation whether it preserves 3-connectedness in all embedded graphs or not.

Keywords: local symmetry-preserving operations; lopsp-operations; topological graph theory; connectivity

MSC(2010): 05C10, 05C40

References

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Isomorphism-free enumeration of polycyclic hydrocarbons using blueprint generation

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Joint work with: Gunnar Brinkmann

Abstract

There exists a number of general approaches for the isomorphism-free enumeration of combinatorial structures, including *orderly generation*, the *homeomorphism-principle*, the *canonical construction path method*, etc. All of them have certain advantages and disadvantages which prohibit using them for every enumeration problem.

A (combinatorial) polycyclic hydrocarbon is a plane graph where every face (except the outer face) is either a hexagon or a pentagon, and every vertex has either degree 2 or 3, the former vertices all lying in the outer face. They can also be thought of as *patches* cut out of fullerenes.

In this talk, we present a novel enumeration technique called *blueprint generation* and apply it to the enumeration of polycyclic hydrocarbons having a fixed number of carbons and hydrogens (vertices of degree 3 or 2 respectively).

Keywords: Structure generation, isomorphism rejection, graph theory

MSC(2010): 05C30, 05C85

Schematic 4-designs

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Joint work with: Vedran Krčadinac

Abstract

A t-(v, k, λ) design is said to be *quasi-symmetric* if it has only two block intersection numbers. It is known that t can be at most 4 [2] and the only quasi-symmetric 4-designs are the derived Witt design 4-$(23, 7, 1)$ and its complement [1]. Regarding designs with three intersection numbers, t can be at most 5 and the only examples are hypothesized to be the Witt design 5-$(24, 8, 1)$ and its complement [3].

We study 4-designs with three intersection numbers. By the Cameron-Delsarte theorem, the blocks form a symmetric association scheme with three classes. This imposes strong restrictions on the parameters of such designs. We calculate the eigenvalues of the association scheme from the design parameters and determine all admissible parameters with at most 1000 points. An infinite family of admissible parameters is discovered.

Keywords: combinatorial design; association shemes

MSC(2010): 05B05, 05E30

References

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Designs in polar spaces

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Joint work with: Michael Kiermaier, Kai-Uwe Schmidt

Abstract

Combinatorial designs have been studied since the 19th century and have famous applications in the design of experiments and in coding theory. 50 years ago, Cameron, Delsarte and Ray-Chaudhury introduced the notion of subspace designs, also known as q-analogs of designs or designs over finite fields. Roughly speaking, q-analogs of objects arise from their combinatorial counterparts by replacing subsets by subspaces and cardinalities by dimensions.

A next natural generalization of subspace designs are designs in finite classical polar spaces. For $t = 1$ and $\lambda = 1$ these objects are known as spreads. For $t > 1$ the first – non-trivial – such designs were found by De Bruyn and Vanhove in 2012, some more designs appeared recently in the PhD thesis of Lansdown.

In this talk we will give an overview on the few known structural results for designs in classical polar spaces and present quite a few new parameters of designs found by computer search.

Keywords: combinatorial designs; subspace designs; finite classical polar space; finite field

MSC(2010): 05B25, 51E20

Density of free codes over finite chain rings

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Joint work with: Eimear Byrne, Anna-Lena Horlemann, Karan Khathuria

Abstract

Ring-linear coding theory has recently gained a lot of (renewed) interest due to possible applications in code-based cryptography. However, many important questions regarding codes over rings have not yet been answered. For example: how likely is it that a random code over a finite chain ring is free? Or how likely is it that a random code achieves the Gilbert-Varshamov bound? In this talk we aim at giving answers to such questions by computing the density of free codes over finite chain rings. We focus on the asymptotics with respect to different parameters (such as the code length or the residue field size), which will give completely different answers. Finally, we remark that these computations have a surprising but interesting intersection with number theory, namely with the Gordon-Andrews identity.

Keywords: Ring-Linear Codes, q-series, Densities

MSC(2010): 11T71, 11P84, 94B65

Packings and Steiner systems in polar spaces

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Joint work with: Kai-Uwe Schmidt

Abstract

A finite classical polar space of rank n consists of the totally isotropic subspaces of a finite vector space equipped with a nondegenerate form such that n is the maximal dimension of such a subspace. A t-Steiner system in a finite classical polar space of rank n is a collection Y of totally isotropic n-spaces such that each totally isotropic t-space is contained in exactly one member of Y. Nontrivial examples are known only for $t = 1$ and $t = n - 1$. We give an almost complete classification of such t-Steiner systems, showing that such objects can only exist in some corner cases. This classification result arises from a more general result on packings in polar spaces, which we obtain by studying the association scheme arising from polar spaces and applying the powerful linear programming method from Delsarte.

Keywords: polar spaces, association schemes, coding theory, linear programming

MSC(2010): 05E30, 51E23, 33C80

Hamiltonian cycles and 1-factors in 5-regular graphs

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Joint work with: Nico Van Cleemput

Abstract

This talk, based on [1], revolves around proper edge-colourings of regular graphs in which certain colour pairs form hamiltonian cycles—such a pair is called *perfect*. We will be particularly interested in the 5-regular case. We begin by presenting a theorem which solves Kotzig's problem asking whether planar 5-regular graphs exist admitting an edge-colouring in which all ten pairs are perfect. In fact, we show that the number of solutions to Kotzig's problem grows at least exponentially. In the second part of the talk, we focus on counting edge-colourings with a certain number of perfect pairs in planar 4-connected 5-regular graphs.

Keywords: Planar; regular; 1-factor; edge-colouring; hamiltonian

MSC(2010): 05C45, 05C07, 05C10, 05C15, 05C70

References

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Disjoint odd circuits in a bridgeless cubic graph can be quelled by a single perfect matching

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Joint work with: František Kardoš and Edita Máčajová

Abstract

Let G be a bridgeless cubic graph. The Berge–Fulkerson Conjecture (1970s) states that G admits a list of six perfect matchings such that each edge of G belongs to exactly two of these perfect matchings. If answered in the affirmative, two other recent conjectures would also be true: the Fan–Raspaud Conjecture (1994), which states that G admits three perfect matchings such that every edge of G belongs to at most two of them; and a conjecture by Mazzuoccolo (2013), which states that G admits two perfect matchings whose deletion yields a bipartite subgraph of G. It can be shown that given an arbitrary perfect matching of G, it is not always possible to extend it to a list of three or six perfect matchings satisfying the statements of the Fan–Raspaud and the Berge–Fulkerson conjectures, respectively. In this talk, we show that given any 1^+-factor F (a spanning subgraph of G such that its vertices have degree at least 1) and an arbitrary edge e of G, there always exists a perfect matching M of G containing e such that $G \setminus (F \cup M)$ is bipartite. Our result implies Mazzuoccolo's conjecture, but not only. It also implies that given any collection of disjoint odd circuits in G, there exists a perfect matching of G containing at least one edge of each circuit in this collection.

Keywords: factor, perfect matching, snark, S_4-Conjecture, Fan–Raspaud Conjecture, Berge–Fulkerson Conjecture

MSC(2010): 05C15, 05C70

Linear sets defined by few points

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Joint work with: Vito Napolitano, Olga Polverino and Paolo Santonastaso

Abstract

A point set L of $\Lambda = \mathrm{PG}(V, \mathbb{F}_{q^n}) = \mathrm{PG}(r-1, q^n)$ is said to be an $\mathbb{F}_q$**-linear set** of Λ of rank k if it is defined by the non-zero vectors of a k-dimensional $\mathbb{F}_q$-vector subspace U of V, i.e.

$$L = L_U := \{\langle \mathbf{u} \rangle_{\mathbb{F}_{q^n}} : \mathbf{u} \in U \setminus \{\mathbf{0}\}\}.$$

For any subspace $S = \mathrm{PG}(Z, \mathbb{F}_{q^n})$ of Λ, the **weight** of S in L_U is defined as $w_{L_U}(S) = \dim_{\mathbb{F}_q}(U \cap Z)$. In this talk we will see some results on linear sets L_U in $\mathrm{PG}(r-1, q^n)$ of rank $k \leq (r-1)n$ containing r independent points $P_1, \ldots, P_r$ such that

$$w_{L_U}(P_1) + \ldots + w_{L_U}(P_r) = k.$$

We will see some characterizations of these linear sets, bounds on their rank and constructions, using multi-orbit cyclic subspace codes. Then we will give a polynomial representation of those linear sets having rank n and we will see the dual of these linear sets, which have few intersection numbers with respect to the hyperplanes. Therefore they also define linear rank metric codes with few weights. The talk is based on the two papers [1, 2].

Keywords: Linear set; subspace codes; rank metric code.

MSC(2010): 05B25, 51E20

References

[1] V. Napolitano, O. Polverino, P. Santonastaso, F. Zullo. "*Linear sets on the projective line with complementary weights*", arXiv::2107.10641, 2021.

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Ortho-Grassmann graphs of finite-rank self-adjoint operators

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Joint work with: Mark Pankov and Krzysztof Petelczyc

Abstract

In a complex Hilbert space H we examine the ortho-Grassmann graph $\Gamma_k^\perp(H)$ whose vertices are k-dimensional subspaces of H and two such subspaces are connected by an edge if they are compatible and adjacent (see [2]). We prove that every automorphism of $\Gamma_k^\perp(H)$ is induced by a unitary or anti-unitary operator, except the case $\dim H = 2k$. If $\dim H = 2k \geq 6$, then additionally, compositions of such automorphisms and the orthocomplementary map are possible. The statement fails for $\dim H = 2k = 4$.

Our result can be expressed in terms of projections of rank k. Applying a characterization of adjacency in terms of ortho-adjacency and an analogue of Chow's theorem for conjugacy classes of finite-rank self-adjoint operators proved in [1], we extend our result on generalised ortho-Grassmann graphs associated to such conjugacy classes.

Keywords: Grassmann graph; graph automorphism; finite rank self-adjoint operator; commutativity

MSC(2010): 05E18, 47B15, 81P10

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